Math 340
HW Solutions

2.1 #4, 8
2.2 #6, 14, 36, 42

2.1 #4

$f(x,y) = \ln(x+y)$

Domain: $\{(x,y) \in \mathbb{R}^2 \mid x+y>0\}$ or $y>-x$.

Range: $\mathbb{R}$ or $(-\infty, \infty)$

Note: $\ln x$ has a domain of $\{x \mid x>0\}$ or $(0,\infty)$ but for $\ln(x+y)$, the $x+y>0$ to make $\ln(x+y)$ defined

2.1 #8

$f: \mathbb{R}^3 \to \mathbb{R}^3 \quad f(\vec{x}) = \vec{x} + 3\vec{j}$ see page 82.

$\vec{x} \in \mathbb{R}^3$ so $\vec{x} = (x_1, x_2, x_3)$

$$f(\vec{x}) = f(x_1, x_2, x_3) = (f_1(x_1,x_2,x_3), f_2(x_1,x_2,x_3), f_3(x_1,x_2,x_3))$$
$$= (x_1, x_2+3, x_3).$$

2.2 #6

$\{(x,y,z) \in \mathbb{R}^3 \mid 1 < x^2+y^2 < 4\}$ is open.

2.2 #14

$$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$$

test $x=y$ $\quad \displaystyle\lim_{x\to 0} \frac{x^2}{x^2+x^2} = \lim_{x\to 0} \frac{x^2}{2x^2} = \frac{1}{2}.$

test $x=0$ $\quad \displaystyle\lim_{y\to 0} \frac{0}{y^2} = 0$

Since the limits along these two paths are different, the limit does not exist!

2.2 #36

$g(x,y) = \dfrac{x^2-y^2}{x^2+1}$ domain is $\mathbb{R}^2$ since $x^2+1 \neq 0 \; \forall x$.

Then, from page 106, we can see that

$\dfrac{x^2-y^2}{x^2+1}$ is continuous if x^2-y^2 and x^2+1 are continuous

since products/quotients of continuous functions are continuous.

x^2-y^2 is continuous if x^2 and $-y^2$ are continuous

$x^2 = x \cdot x$ and $-y^2 = -y \cdot y$

x^2+1 is continuous if x^2 and 1 are continuous.

$f(x)=x \quad f(a)=a \qquad f(x)=1 \quad f(a)=1$

$\lim_{x\to a} x = a \quad \forall a \qquad \lim_{x\to a} 1 = 1 \quad \forall a.$ etc.

By breaking down the functions to their basic components, we can use rules of continuity to determine whether a function is continuous.

(this is much more than you needed to say.)

2.2 #42

$$g(x,y) = \begin{cases} \dfrac{x^3+xy^2+2x^2+2y^2}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ c & (x,y) = (0,0) \end{cases}$$

$$\lim_{(x,y)\to(0,0)} \frac{x^3+xy^2+2x^2+2y^2}{x^2+y^2}$$

for $y=0$ $\quad \lim_{x\to 0} \dfrac{x^3+2x^2}{x^2} = \lim_{x\to 0} \dfrac{x^2(x+2)}{x^2} = \lim_{x\to 0} x+2 = 2.$

Since the problem implies the limit exists, we can stop here.

For example's sake!

for $x=y$ $\quad \lim_{x\to 0} \dfrac{x^3+x^3+2x^2+2x^2}{2x^2+2x^2} = \lim_{x\to 0} \dfrac{2x^2(x+2)}{2x^2} = \lim_{x\to 0} x+2 = 2.$

So this tells us that $g(0,0) = 2 = c$ if $g(x,y)$ is continuous.