11/8/06 Math 340 Quiz **Name:**

Find $\int\int_D x^2\,dA$ where D is the region in the first quadrant bounded by the ellipse $(x/2)^2 + (y/3)^2 = 1$ and the x and y axes.

Solution

This is similar to problem 11 in 5.8.

Let $u = x/2$ and $v = y/3$. Then $x = 2u$, $y = 3v$ so $\partial(x,y)/\partial(u,v) = 6$. So $\int\int_D x^2\,dA = \int\int_E (2u)^2\,6\,dudv$ where E is the quarter circle bounded by $u^2 + v^2 = 1$ and the u and v axes. It is best to change this to polar coordinates,

$$\int\int_E (2u)^2\,6\,dudv = \int\int_E 24u^2\,dudv = \int_0^{\pi/2}\int_0^1 24r^3\cos^2\theta\,drd\theta$$

$$= \int_0^{\pi/2} 6r^4\cos^2\theta\Big]_0^1 d\theta = \int_0^{\pi/2} 6\cos^2\theta\,d\theta = \int_0^{\pi/2} 3\cos(2\theta) + 3\,d\theta$$

$$= 1.5\sin(2\theta) + 3\theta\Big]_0^{\pi/2} = 3\pi/2$$