11/15/06 Math 340 Quiz Name:

Let C be the curve parametrized by $\mathbf{x}(t) = t^2\mathbf{i} + \frac{4}{3}t^{3/2}\mathbf{j} + t\mathbf{k}$ for $1 \le t \le 4$. Find:

velocity ____________________

acceleration ____________________

speed ____________________ (you would be wise to simplify this).

a_{norm} ____________________

curvature ____________________

$\mathbf{T}$ ____________________

the length of C ____________________

Solution

Velocity $= x'(t) = 2t\mathbf{i} + 2t^{1/2}\mathbf{j} + \mathbf{k}$.

Acceleration $= x''(t) = 2\mathbf{i} + t^{-1/2}\mathbf{j}$.

The speed $= ||x'(t)|| = \sqrt{4t^2 + 4t + 1} = 2t + 1$.

$a_{tang} = 2$, the derivative of the speed. So $a_{norm} = \sqrt{||a||^2 - a_{tan}^2} = \sqrt{2^2 + (t^{-1/2})^2 - 2^2} = t^{-1/2}$. You could also use the formula $||v \times a||/||v|| = ||t^{-1/2}\mathbf{i} + 2\mathbf{j} - 2t^{1/2}\mathbf{k}||/(2t+1) = \sqrt{1/t + 4 + 4t}/(2t+1) = t^{-1/2}$.

The curvature is $\kappa = a_{norm}/\text{speed}^2 = t^{-1/2}(2t+1)^{-2}$.

The unit tangent vector is $\mathbf{T} = x'(t)/||x'(t)|| = \frac{2t}{2t+1}\mathbf{i} + \frac{2t^{1/2}}{2t+1}\mathbf{j} + \frac{1}{2t+1}\mathbf{k}$.

The length of C is the integral of the speed, $\int_1^4 2t + 1\, dt = t^2 + t\Big]_1^4 = 18$.