11/22/06 Math 340 Quiz **Name:**

1. Find the integral $\int_C x\,dy$ where C is portion of the circle $(x-2)^2+y^2=4$ in the first quadrant, oriented by starting at $(0,0)$ and ending at $(4,0)$.

You can parameterize C by $r(t)=(2+2\cos t)\mathbf{i}+2\sin t\mathbf{j}$, $0\le t\le\pi$. This parameterization has the wrong orientation however, so

$$\int_C x\,dy = -\int_0^\pi (2+2\cos t)2\cos t\,dt = -\int_0^\pi 4\cos t+4\cos^2 t\,dt$$

$$= -\int_0^\pi 4\cos t+2(1+\cos(2t))\,dt = -(4\sin t+2t+\sin(2t))\Big]_0^\pi$$

$$= -(4\sin\pi+2\pi+\sin(2\pi))+(4\sin 0+0+\sin(0)) = -2\pi$$

Here's another way to do this. Let C_1 be the line segment from $(0,0)$ to $(4,0)$, parameterized by $r_1(t)=4t\mathbf{i}$, $0\le t\le 1$. Then $\int_{C_1} x\,dy=\int_0^1 4t\cdot 0dt = 0$. But by Green's theorem, $\int_{C_1-C} x\,dy=\int\int_D 1\,dA=(1/2)\pi 2^2=2\pi$ where D is the region enclosed by C and C_1 which is half a disc of radius 2. So $2\pi=\int_{C_1-C} x\,dy=\int_{C_1} x\,dy-\int_C x\,dy = 0-\int_C x\,dy$. So $\int_C x\,dy=-2\pi$.

2. Is the vector field $(y^2,2xy)$ conservative? Find $\int_C y^2\,dx+2xy\,dy$ where C is the curve which goes in a straight line from $(1,2)$ to $(1,0)$, then a quarter circle to $(0,1)$, then on the parabola $y=1-x^2$ to $(2,-3)$.

If $\partial g/\partial x=y^2$ then $g(x,y)=xy^2+C(y)$. Plug this in to $\partial g/\partial y=2xy$ and we get $2xy+C'(y)=2xy$ so $C(y)$ is a constant. So $(y^2,2xy)=\text{grad}(xy^2)$ and is conservative. $\int_C y^2\,dx+2xy\,dy=g(2,-3)-g(1,2)=18-4=14$.