11/29/06 Math 340 Quiz **Name:**

Let Σ be the part of the plane $z = 3x - y + 2$ inside the cylinder $x^2 + y^2 = 4$.

a) Find the area of Σ.

b) Find the flux integral $\int\int_\Sigma \mathbf{F} \cdot \mathbf{n}\, dS$ where $\mathbf{F}(x, y, z) = y\mathbf{i} + 2x\mathbf{j} + 4z\mathbf{k}$ and we orient Σ pointing upwards.

Parameterize Σ by $X(x, y) = (x, y, 3x - y + 2)$ for $x^2 + y^2 \leq 4$. Then

$$\mathbf{n}\, dS = X_x \times X_y\, dydx = -3\mathbf{i} + \mathbf{j} + \mathbf{k}\, dydx$$

and

$$dS = \sqrt{3^2 + 1^2 + 1^2}\, dydx = \sqrt{11}\, dydx$$

The area is

$$\int\int_\Sigma dS = \int_0^{2\pi} \int_0^2 \sqrt{11} r\, drd\theta = \sqrt{11}\pi 2^2 = 4\sqrt{11}\pi$$

The flux integral is

$$\int_0^{2\pi} \int_0^2 (y, 2x, 4z) \cdot (-3, 1, 1)\, r\, drd\theta$$

$$= \int_0^{2\pi} \int_0^2 -3r\sin\theta + 2r\cos\theta + 4(3r\cos\theta - r\sin\theta + 2)\, r\, drd\theta$$

$$= \int_0^{2\pi} \int_0^2 -7r^2\sin\theta + 14r^2\cos\theta + 8r\, drd\theta$$

$$= \int_0^{2\pi} -7/3r^3\sin\theta + 14/3r^3\cos\theta + 4r^2\Big]_0^2\, d\theta$$

$$= \int_0^{2\pi} -56/3\sin\theta + 112/3\cos\theta + 16\, d\theta$$

$$= 56/3\cos\theta + 112/3\sin\theta + 16\theta\Big]_0^{2\pi} = 32\pi$$