12/06/06 Math 340 Quiz **Name:**

Let Σ be the part of the plane $3x-y+2z=4$ inside the cylinder $x^2+y^2=9$. Let C be the boundary of Σ, oriented clockwise as viewed from above. Use Stokes' theorem to compute $\int_C \mathbf{F}\cdot T\,ds$ where $\mathbf{F}(x,y,z)=(y+ze^{xz})\mathbf{i}+xe^{xz}\mathbf{k}$.

The curl of F is $-\mathbf{k}$. The surface is given by $z=2-1.5x+.5y$ and is oriented downwards, so $\mathbf{n}\,dS=-(1.5,-.5,1)dxdy$. Stokes' theorem says

$$\int_C \mathbf{F}\cdot T\,ds = -\int\int_\Sigma curl\mathbf{F}\cdot\mathbf{n}\,dS$$

$$= \int\int_D -\mathbf{k}\cdot(-1.5,.5,-1)\,dA = \int\int_D 1dA$$

So $\int_C \mathbf{F}\cdot T\,ds$ is the area of the shadow D of Σ which is a disc of radius 3. So $\int_C \mathbf{F}\cdot T\,ds = 9\pi$.