1/31/07 Math 341 Quiz **Name:**

Let α be the basis $\{(1,2),(3,4)\}$ of $\mathbb{R}_{1\times 2}$. Let $\tau\colon \mathbb{R}_{1\times 2} \to \mathbb{R}_{1\times 2}$ be the linear transformation $\tau(a,b) = (b, 2a)$.

1) Find $Mtx_{\alpha,\alpha}(\tau)$.
   $\tau(1,2) = (2,2) = -(1,2) + (3,4)$ so the first column is $(-1\ 1)^T$.
   $\tau(3,4) = (4,6) = (1,2) + (3,4)$ so the second column is $(1\ 1)^T$.
   So $Mtx_{\alpha,\alpha}(\tau) = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}$.
2) Determine whether or not τ is one to one (and give sufficient reasons).
   τ is one to one since $Mtx_{\alpha,\alpha}(\tau)$ has rank 2 = the number of columns. Another reason is that the columns of $Mtx_{\alpha,\alpha}(\tau)$ are linearly independent. Many more reasons are possible.
3) Determine whether or not τ is onto (and give sufficient reasons).
   τ is onto since $Mtx_{\alpha,\alpha}(\tau)$ has rank 2 = the number of rows. Many more reasons are possible.