2/21/07 Math 341 Quiz **Name:**

Find and classify (relative max, relative min, saddle, or degenerate) all the critical points of $f(x, y) = x^3 - 6xy - y^2 + 3$.

$0 = \nabla f = (3x^2 - 6y,\ -6x - 2y)$ so $y = -3x$, so $3x^2 = 6y = -18x$ so $x = 0$ or $x = -6$. In case $x = 0$ then $y = -3x = 0$ so one critical point is $(0, 0)$. In case $x = -6$ then $y = -3x = 18$ so the other critical point is $(-6, 18)$. The Hessian is $\begin{bmatrix} 6x & -6 \\ -6 & -2 \end{bmatrix}$. Its determinent is $D = -12x - 36$. The determinent is -36 at $(0, 0)$ so $(0, 0)$ is a saddle point. The determinent is 36 at $(-6, 18)$ and $\partial^2 f/\partial x^2 = -36 < 0$ so $(-6, 18)$ is a local maximum.

Of course you could also just find the characteristic values of the Hessian at each point. At $(0, 0)$ the Hessian is $\begin{bmatrix} 0 & -6 \\ -6 & -2 \end{bmatrix}$ which has characteristic polynomial $\lambda^2 + 2\lambda - 36$ which has roots $-1 \pm \sqrt{37}$ or roughly 5 and -7. At $(-6, 18)$ the Hessian is $\begin{bmatrix} -36 & -6 \\ -6 & -2 \end{bmatrix}$ which has characteristic polynomial $\lambda^2 + 38\lambda + 36$ which has roots $-19 \pm \sqrt{325}$ or roughly -1 and -37.