2/7/07 Math 341 Quiz Name:

Find the characteristic polynomial, characteristic values, and characteristic vectors of the matrix $A = \begin{bmatrix} 2 & 3 \\ 2 & 1 \end{bmatrix}$. If possible, find a matrix P so that $P^{-1}AP$ is diagonal, if not possible, explain why not.

$\det(\lambda I - A) = \det \begin{bmatrix} \lambda - 2 & -3 \\ -2 & \lambda - 1 \end{bmatrix} = (\lambda - 2)(\lambda - 1) - 6 = \lambda^2 - 3\lambda - 4 = (\lambda - 4)(\lambda + 1) =$ the characteristic polynomial.

So the characteristic values are $\lambda = 4$ and $\lambda = -1$.

The characteristic vectors for $\lambda = 4$ are $NS(A - 4I) = NS \begin{bmatrix} -2 & 3 \\ 2 & -3 \end{bmatrix} = Span \begin{bmatrix} 3 \\ 2 \end{bmatrix}$. To be precise, the characteristic vectors for $\lambda = 4$ are all nonzero multiples of $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$.

The characteristic vectors for $\lambda = -1$ are $NS(A + I) = NS \begin{bmatrix} 3 & 3 \\ 2 & 2 \end{bmatrix} = Span \begin{bmatrix} 1 \\ -1 \end{bmatrix}$. To be precise, the characteristic vectors for $\lambda = -1$ are all nonzero multiples of $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

Let $P = \begin{bmatrix} 3 & 1 \\ 2 & -1 \end{bmatrix}$. Then we are guaranteed that $P^{-1}AP = \begin{bmatrix} 4 & 0 \\ 0 & -1 \end{bmatrix}$. You could switch the columns of P which would switch 4 and -1. You could also multiply a column of P by a nonzero scalar and get the same result.