

Find all solutions of each of the following:

1. $y'' - 2y' + 10y = 0$.

The roots of $r^2 - 2r + 2$ are $r = 1 \pm 3i$ so all solutions are $y = c_1 e^t \cos 3t + c_2 e^t \sin 3t$.

2. $y'' - 6y' + 8y = 0$

The roots of $r^2 - 6r + 8$ are $r = 2, 4$ so all solutions are $y = c_1 e^{2t} + c_2 e^{4t}$.

3. $y'' + 4y' + 4y = 0, y(0) = 2, y'(0) = 5$.

$r^2 + 4r + 4 = (r + 2)^2$ has a double root $r = -2$ so all solutions are of the form $y = c_1 e^{-2t} + c_2 t e^{-2t}$.

Then $y' = -2c_1 e^{-2t} + c_2(1 - 2t)e^{-2t}$ so

$$2 = y(0) = c_1$$

$$5 = y'(0) = -2c_1 + c_2$$

So $c_2 = 5 + 2c_1 = 9$. So the unique solution is $y = (2 + 9t)e^{-2t}$.
