3/28/07 Math 341 Quiz **Name:**

1. Find all solutions of $y'' + 4y' + 8y = 20\cos t$, $y(0) = 3$, $y'(0) = 6$.

Try $y = a\cos t + b\sin t$. Then $y' = -a\sin t + b\cos t$ and $y'' = -a\cos t - b\sin t$. So $y'' + 4y' + 8y = -a\cos t - b\sin t - 4a\sin t + 4b\cos t + 8a\cos t + 8b\sin t = (7a + 4b)\cos t + (7b - 4a)\sin t$. So $7a + 4b = 20$ and $7b - 4a = 0$, so $b = 4a/7$ and $20 = 7a + 4b = 65a/7$ so $a = 140/65 = 28/13$ and $b = 4a/7 = 16/13$. So the general solution is $y = 28/13\cos t + 16/13\sin t + c_1e^{-2t}\cos 2t + c_2e^{-2t}\sin 2t$. We have $3 = y(0) = 28/13 + c_1$ so $c_1 = 11/13$. We have $y' = -28/13\sin t + 16/13\cos t - 2c_1e^{-2t}\cos 2t - 2c_1e^{-2t}\sin 2t - 2c_2e^{-2t}\sin 2t + 2c_2e^{-2t}\cos 2t$. So $6 = y'(0) = 16/13 - 2c_1 + 2c_2 = -6/13 + 2c_2$ so $c_2 = 42/13$. So there is only one solution, $y = (28\cos t + 16\sin t + 11e^{-2t}\cos 2t + 42e^{-2t}\sin 2t)/13$.

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2. For each of the following, write down the form of a solution you would use in the method of judicious guessing. Do not solve for the coefficients.

a) $y'' + 4y' + 3y = 3te^{-t}$.

$y = (at + b)te^{-t}$.

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b) $y'' + 2y' + y = 3te^{-t}$.

$y = (at + b)t^2e^{-t}$.

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c) $y'' - 4y' + 3y = 3te^{-t}$.

$y = (at + b)e^{-t}$.
