3/7/07 Math 341 Quiz Name:

Find all solutions of each of the following:

1. $y' + 4ty = 2t$

Multiply by $e^{\int 4t\,dt} = e^{2t^2}$. We then get $(e^{2t^2}y)' = e^{2t^2}y' + 4te^{2t^2}y = 2te^{2t^2}$. Integrating we get $e^{2t^2}y = e^{2t^2}/2 + c$ so $y = .5 + ce^{-2t^2}$ is the general solution.

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2. $(y + 2x^2 - x\cos(xy))y' + 4xy + e^x - y\cos(xy) = 0$, $y(1) = 0$.

We hope this is exact and so try to solve

$$\partial\phi/\partial x = 4xy + e^x - y\cos(xy)$$

$$\partial\phi/\partial y = y + 2x^2 - x\cos(xy)$$

From the first equation we get $\phi(x, y) = 2x^2y + e^x - \sin(xy) + C(y)$ and plugging this into the second equation we get $2x^2 + 0 - x\cos(xy) + C'(y) = y + 2x^2 - x\cos(xy)$. So $C'(y) = y$ and thus $C(y) = y^2/2 + D$. We only need one ϕ so we may set $D = 0$. So the general solution is $\phi(x, y) = E$ or $2x^2y + e^x - \sin(xy) + y^2/2 = E$. Plugging in the initial values $x = 1$, $y = 0$ we get $21^20 + e^1 - \sin(0) + 0^2/2 = E$ so $E = e$. Thus there is only one solution, the implicit function

$$2x^2y + e^x - \sin(xy) + y^2/2 = e$$

near $x = 1$, $y = 0$.
