4/11/07 Math 341 Quiz   Name:

Solve $y'''' - 3y''' + 2y'' = 12e^{-t} + 24t^2 - 84t + 42 - 10\cos t$, $y(0) = 8$, $y'(0) = 1$, $y''(0) = 2$, $y'''(0) = -4$.

$r^4 - 3r^3 + 2r^2 = (r-2)(r-1)r^2$ so homogeneous solutions are $c_1e^{2t} + c_2e^t + c_3 + c_4t$. Use judicious guessing. For $12e^{-t}$ try $y = ae^{-t}$ then $y'''' - 3y''' + 2y'' = ae^{-t} + 3ae^{-t} + 2ae^{-t} = 6ae^{-t}$ so let $a = 2$ and $y_1 = 2e^{-t}$. For $24t^2 - 84t + 42$ try $y = (at^2 + bt + c)t^2 = at^4 + bt^3 + ct^2$. Then $y' = 4at^3 + 3bt^2 + 2ct$, $y'' = 12at^2 + 6bt + 2c$, $y''' = 24at + 6b$, $y'''' = 24a$. Then $y'''' - 3y''' + 2y'' = 24a - 72at - 18b + 24at^2 + 12bt + 4c = 24t^2 - 84t + 42$. So $24a = 24$, $-72a + 12b = -84$, and $24a - 18b + 4c = 42$. So $a = 1$, $b = -1$, $c = 0$ and $y_2 = t^4 - t^3$. For $-10\cos t$ try $y = a\cos t + b\sin t$. Then $y'''' - 3y''' + 2y'' = a\cos t + b\sin t - 3(a\sin t - b\cos t) + 2(-a\cos t + -b\sin t) = (3b - a)\cos t - (3a + b)\sin t$ so $b = -3a$ and $-10 = 3b - a = -10a$ so $a = 1$, $b = -3$ and $y_3 = \cos t - 3\sin t$. Combining all this we see the general solution is

$$y = 2e^{-t} + t^4 - t^3 + \cos t - 3\sin t + c_1e^{2t} + c_2e^t + c_3 + c_4t$$

We have $8 = y(0) = 3 + c_1 + c_2 + c_3$, $1 = y'(0) = -2 - 3 + 2c_1 + c_2 + c_4$, $2 = y''(0) = 2 - 1 + 4c_1 + c_2$, and $-4 = y'''(0) = -2 - 6 + 3 + 8c_1 + c_2$. From the last two equations, $1 = 4c_1 + c_2$ and $1 = 8c_1 + c_2$ from which we get $c_1 = 0$, $c_2 = 1$. Then $c_3 = 5 - c_1 - c_2 = 4$ and $c_4 = 6 - 2c_1 - c_2 = 5$. So the solution is

$$y = 2e^{-t} + t^4 - t^3 + \cos t - 3\sin t + e^t + 4 + 5t$$

As a check we have $y' = -2e^{-t} + 4t^3 - 3t^2 - \sin t - 3\cos t + e^t + 5$ so $y'(0) = -2 - 3 + 1 + 5 = 1$, $y'' = 2e^{-t} + 12t^2 - 6t - \cos t + 3\sin t + e^t$ so $y''(0) = 2 - 1 + 1 = 2$, and $y''' = -2e^{-t} + 24t - 6 + \sin t + 3\cos t + e^t$ so $y'''(0) = -2 - 6 + 3 + 1 = -4$