

1. Find the Laplace transform of $f(t) = \begin{cases} t^2 & \text{if } t < 2 \\ 4 & \text{if } t \geq 2 \end{cases}$.
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We have $f(t) = t^2 + H_2(t)(4 - t^2) = t^2 + H_2(t)(-(t - 2)^2 - 4(t - 2))$. So $\mathcal{L}(f) = 2/s^3 + e^{-2s}(-2/s^3 - 4/s^2)$.

2. Find all solutions to $y'' + 2y' = \delta(t - 2)$, $y(0) = 1$, $y'(0) = 2$.
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$e^{-2s} = \mathcal{L}(\delta(t-2)) = \mathcal{L}(y'' + 2y') = s^2Y - s - 2 + 2(sY - 1) = s(s+2)Y - s - 4$. So $Y = (s + 4)/(s(s + 2)) + e^{-2s}/(s(s + 2))$. We want $1/(s(s + 2)) = a/s + b/(s + 2)$ so $1 = a(s + 2) + bs$ and so $a = 1/2$ and $b = -1/2$. Then also $(s + 4)/(s(s + 2)) = 1/(s + 2) + 4/(s(s + 2)) = 1/(s + 2) + 2/s - 2/(s + 2) = 2/s - 1/(s + 2)$. So the solution is $y = 2 - e^{-2t} + H_2(t)(.5 - .5e^{-2(t-2)})$.
