5/2/07 Math 341 Quiz Name:

1. Consider the system $x' = -3x - 2y - z$, $y' = 2y - z$, $z' = -z$. Determine the stability (assymptotically stable, stable but not asssymptotically stable, or unstable) of the critical point 0. Give a short reason for your answer.

The eigenvalues are -3,2,-1. Since at least one eigenvector is positive, this critical point is unstable.

---

2. Find all critical points of the system $x' = -x - 6y$, $y' = x - x^2$. Draw a picture of the solution curves to the system near each critical point. Incorporate information from the eigenvectors if appropriate. Determine the stability of each critical point.

At a critical point, $x - x^2 = 0$ so $x = 0, 1$. Also $-x - 6y = 0$ so the critical points are $(0, 0)$ and $(1, -1/6)$. The linearized system has matrix $\begin{pmatrix} -1 & -6 \\ 1 - 2x & 0 \end{pmatrix}$. At $(0, 0)$ this matrix is $\begin{pmatrix} -1 & -6 \\ 1 & 0 \end{pmatrix}$ with characteristic polynomial $x^2 + x + 6$ which has two complex roots with negative real part. So this critical point is assymptotically stable and spirals in to the origin, Since $x' < 0$ on the positive y axis, we know it looks like figure 6b on page 424.

At $(1, -1/6)$ the matrix of the linearized system is $\begin{pmatrix} -1 & -6 \\ -1 & 0 \end{pmatrix}$ with characteristic polynomial $x^2 + x - 6$ so the eigenvalues are 2 and -3. So this is a saddle as in figure 5 on page 423 (but the straight lines in the picture might curve a bit). The curves ℓ_1 and ℓ_1' in the figure are tangent at the origin to an eigenvector of -3 which is $(3, 1)$. The curves ℓ_2 and ℓ_2' in the figure are tangent at the origin to an eigenvector of 2 which is $(-2, 1)$.
