

### Math 241 Exam 2 Sample 3

**Directions:** Do not simplify unless indicated. No calculators are permitted. Show all work as appropriate for the methods taught in this course. Partial credit will be given for any work, words or ideas which are relevant to the problem.

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#### Please put problem 1 on answer sheet 1

- (a) Find the directional derivative of  $h(x, y) = 3x^2y$  at  $(1, 1)$  in the direction of  $\hat{i} - \hat{j}$ . [8 pts]  
(b) First find the equation of the plane tangent to the surface  $f(x, y) = x^2 + y^2$  at  $(2, 1, 5)$  and then show that the point  $(1, 0, -1)$  is contained within this plane. [12 pts]

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#### Please put problem 2 on answer sheet 2

- (a) Suppose a triangle is growing in size such that the base is growing at 2cm/sec and the height is growing at 3cm/sec. At what rate is the area growing when the base is 10cm and the height is 20cm? [10 pts]  
(b) The function  $f(x, y) = x^2y - 2x^2 - y^2$  has the following: [10 pts]

$$f_{xx}(x, y) = 2y - 4 \quad f_{yy}(x, y) = -2 \quad f_{xy}(x, y) = 2x$$

There are three critical points at  $(0, 0)$ ,  $(2, 2)$  and  $(-2, 2)$ . Categorize each critical point as a relative maximum, relative minimum or saddle point.

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#### Please put problem 3 on answer sheet 3

- (a) Sketch the surface  $z = 9 - \sqrt{x^2 + y^2}$ . Be sure to include some points or tick marks to give a sense of scale/position. [5 pts]  
(b) Sketch the surface  $f(x, y) = x^2 - 4$ . Be sure to include some points or tick marks to give a sense of scale/position. [5 pts]  
(c) Write down the equation for the cylinder of radius 3 which runs along the  $z$ -axis. [5 pts]  
(d) Write down the equation for the paraboloid opening up from the vertex  $(0, 0, 1)$ . [5 pts]

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#### Please put problem 4 on answer sheet 4

- Find the extreme values of  $f(x, y) = (x - 1)^2 + y^2$  subject to the constraints  $x^2 + y^2 \leq 4$  and  $x \geq 0$ . [20 pts]

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#### Please put problem 5 on answer sheet 5

- Use Lagrange multipliers to find the maximum and minimum values of the function  $f(x, y) = xy + 2x$  on the curve  $x^2 + y^2 = 4$ . [20 pts]

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The End