Math 241 Fun Exam 3

Directions: Do not evaluate or simplify unless indicated. No calculators are permitted. Show all work as appropriate for the methods taught in this course. Partial credit will be given for any work, words or ideas which are relevant to the problem. Very few pictures are required! Do not evaluate or simplify unless indicated!

Please put problem 1 on answer sheet 1

1. Consider the region R below $y = 10 - 2x$ and in the first quadrant and the integral $\iint_R x \, dA$.

   (a) Set up the integral as an iterated double integral treating R as vertically simple. [4 pts]

   (b) Set up the integral as an iterated double integral treating R as horizontally simple. [6 pts]

   (c) Set up the integral as an iterated double integral treating R as polar. [10 pts]

Please put problem 2 on answer sheet 2

2. (a) Set up the iterated integral in cylindrical coordinates for the volume of D, the solid above the xy-plane, inside the cylinder $r = 2\cos\theta$, outside the cylinder $r = 1$ and below the paraboloid $z = 9 - x^2 - y^2$. Draw the related region R. [15 pts]

   (b) Sketch the surface parametrized by $\bar{r}(x, \theta) = x\,\hat{\imath} + \cos\theta\,\hat{\jmath} + 3\sin\theta\,\hat{k}$ with $0 \le x \le 2$ and $0 \le \theta \le \pi$. [5 pts]

Please put problem 3 on answer sheet 3

3. (a) Set up the iterated integral in rectangular coordinates for $\iiint_D x \, dV$ where D is the solid in the first octant and below the plane $x + 2y + 2z = 10$. Draw the related region R. [15 pts]

   (b) Write down a parametrization for the portion of the cylinder $x^2 + z^2 = 9$ above the rectangle with corners $(0, 0, 0)$, $(0, 1, 0)$, $(2, 1, 0)$ and $(2, 0, 0)$. [5 pts]

Please put problem 4 on answer sheet 4

4. (a) Set up the iterated integral in spherical coordinates for $\iiint_D y \, dV$ where D is the solid inside the cone $z = \sqrt{3x^2 + 3y^2}$, outside the sphere $x^2 + y^2 + z^2 = 1$, below the plane $z = 2$ and having $y \ge 0$. [10 pts]

   (b) Reparametrize as polar and evaluate the integral $\int_{-1}^{1} \int_{0}^{\sqrt{1-x^2}} e^{x^2+y^2} \, dy \, dx$. [10 pts]

Please put problem 5 on answer sheet 5

5. Perform a change of variables which changes the integral $\iint_R x \, dA$ to an iterated double integral over a rectangle S in the uv-plane. Here R is the region bounded by the lines $y = x - 3$, $y = x + 5$, $y = -3x + 5$ and $y = -3x - 1$. Make sure all your steps are clear and draw both regions R and S. Your final answer should be an iterated integral. [20 pts]

Welcome to the End of the Exam