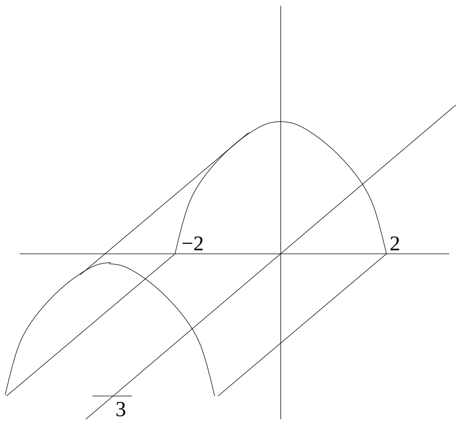
Math 241 Exam 3 Sample 3 Solutions

1. (a) The easiest method would be $\bar{r}(r,\theta) = r\cos\theta\,\hat{\imath} + r\sin\theta\,\hat{\jmath} + (9 - r^2)\,\hat{k}$ with $0 \leq r \leq 3$ and $0 \leq \theta \leq 2\pi$.

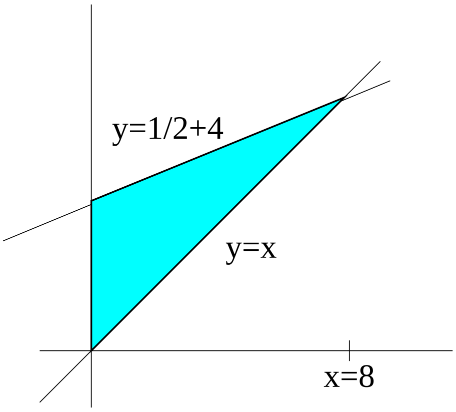
   (b) Since $z = 4 - y^2$ this shape is a parabolic sheet as shown:



   (c) We need to change this to polar first since the integrand is not integrable with respect to y or x. The region R is the quarter disk of radius 1 in the first quadrant and so we have:

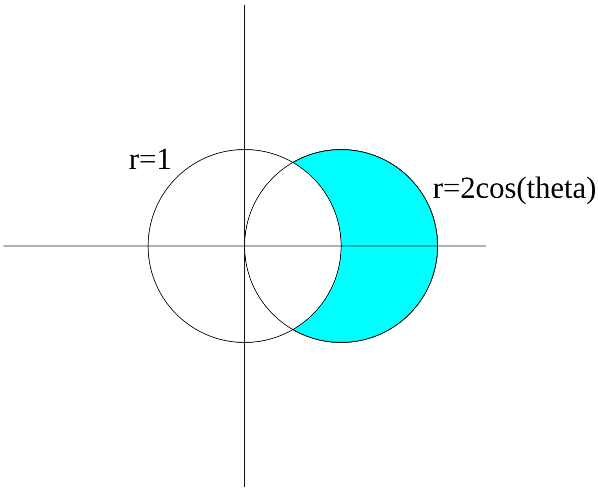
$$\begin{aligned}
\int_0^1 \int_0^{\sqrt{1-x^2}} \sin(x^2+y^2)\ dy\ dx &= \int_0^{\pi/2} \int_0^1 \sin(r^2) r\ dr\ d\theta \\
&= \int_0^{\pi/2} -\frac{1}{2}\cos(r^2)\Big|_0^1\ d\theta \\
&= \int_0^{\pi/2} -\frac{1}{2}(\cos(1) - \frac{1}{2}\cos(0))\ d\theta \\
&= \int_0^{\pi/2} \frac{1}{2}(1-\cos(1))\ d\theta \\
&= \frac{1}{2}\theta(1-\cos(1))\Big|_0^{\pi/2} \\
&= \frac{1}{2}(0)(1-\cos(1)) - \frac{1}{2}(\pi/2)(1-\cos(1))
\end{aligned}$$

2. (a) The picture is:



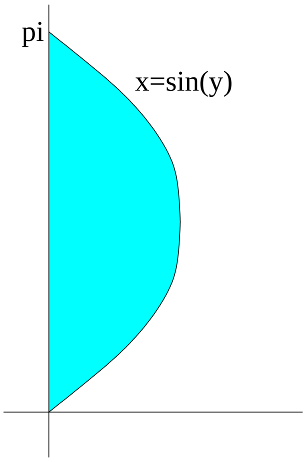
The lines meet when $y = x$ meets $y = \frac{1}{2}x + 4$ at $x = 8$. Thus the integral would be $\int_0^8 \int_x^{\frac{1}{2}x+4} y\ dy\ dx$

(b) The picture is:

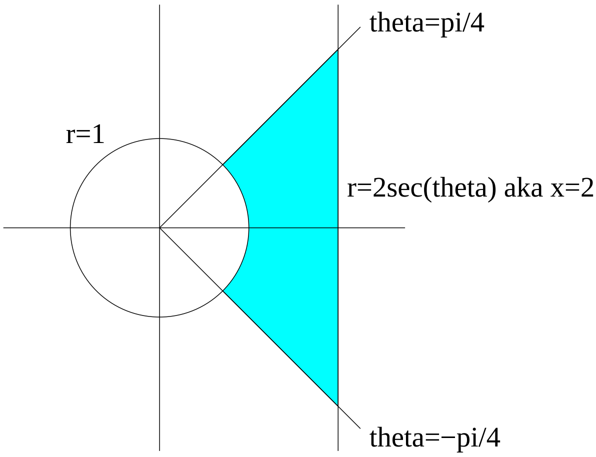


The circles meet when $r = 1$ meets $r = 2\cos\theta$ which is when $\cos\theta = \frac{1}{2}$ or $\theta = \pm\frac{\pi}{3}$. Thus the integral is $\int_{-\pi/3} \pi/3 \int_1^{2\cos\theta} (r\cos\theta)(r\sin\theta) r\ dr\ d\theta$.

3. (a) The picture is:

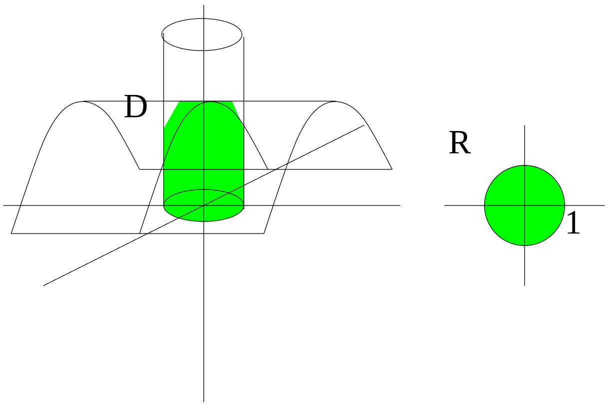


(b) The picture is:



(c) We have $z = 4 - \sqrt{x^2 + y^2} = 4 - \sqrt{r^2} = 4 - r$.

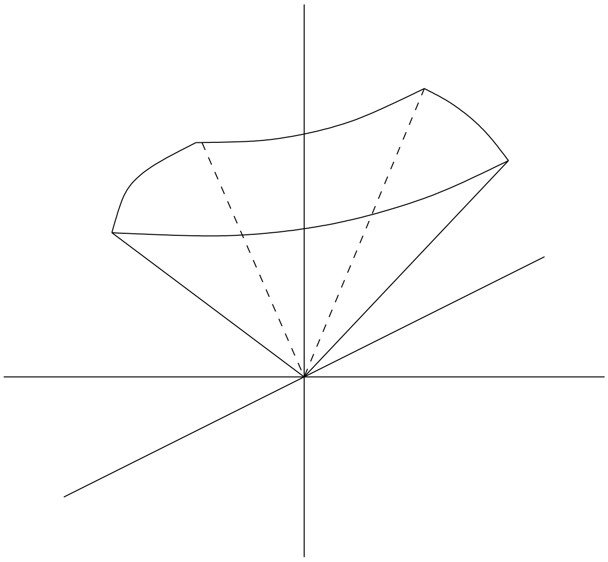
4. (a) The picture is:



The iterated integral is

$$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{0}^{9-x^2} z\ dz\ dy\ dx$$

(b) The picture is:

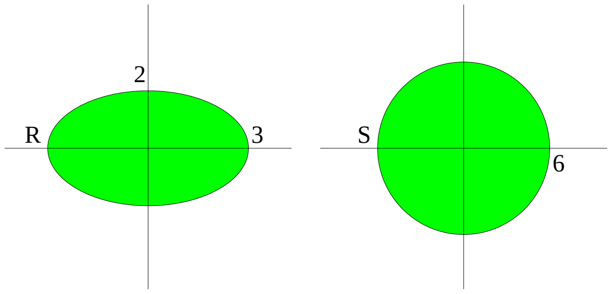


The iterated integral is

$$\int_{0}^{\pi/2}\int_{\pi/6}^{\pi/3}\int_{0}^{3} \rho^2 \sin\phi\ d\rho\ d\phi\ d\theta$$

5. We rewrite the ellipse as $(2x)^2 + (3y)^2 = 36$ and subsitute $u = 2x$ and $v = 3y$. The new region S is then inside the circle $u^2 + v^2 = 26$.

   The pictures are:



   Then $x = u/2$ and $y = v/3$ so that

$$J = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/3 \end{bmatrix} = 1/6$$

   . So we have

$$\iint_R x \; dA = \iint_S \frac{1}{2} u |1/6| \; dA = \frac{1}{12} \iint_S u \; dA$$

   which we then parametrize in polar

$$= \frac{1}{12} \int_0^{2\pi} \int_0^6 r \cos\theta \; r \; dr \; d\theta$$