Math 241 FINAL EXAM
200 points possible

May, 14 2009.

Instructions: Write your name, section number and TA's name on each answer sheet. **Answer each numbered question on a separate answer sheet.** You must show all appropriate work in order to receive credit for an answer. At the end of the exam, arrange the answer sheets in order and write and sign the honor pledge on the first one (only on the first one). **Show all work – No calculators – Good luck!**

1. (25 points) Consider the line through the points $P = (1, 2, 3)$ and $Q = (2, 0, 1)$.
(a) Find the intersection of the line with the xy-plane.
(b) Find the point on the line which is closest to the origin $(0, 0, 0)$.

2. (25 points) An object moves according to the following equation

$$\mathbf{r}(t) = 10 \sin t\, \mathbf{i} + 8 \cos t\, \mathbf{j} + (6 + 6 \cos t)\, \mathbf{k}.$$

(a.) Assuming that the motion starts at $t = 0$, determine how far the object has travelled when $t = 2\pi$.
(b.) Find the tangential and normal components a_T and a_N of the acceleration at $t = \pi/2$.

3. (25 points) Consider the surface S given by the equation

$$x + y + z^2 + xy - x^2 - y^2 = 1.$$

Find the equation of the tangent plane to the surface S at the point $(1, 0, -1)$. Give the equation in the form $ax + by + cz = d$.

4. (25 points) Find and classify each critical points of the following function as a local minimum, a local maximum, or a saddle point

$$f(x, y) = x^4 - 2x^2 + y^2 - 2.$$

Turn the page please

5. (25 points) Let R be the region bounded by the graphs of the functions $y = 4x^2$ and $y = 16x$.
Using Green's theorem evaluate the following line integral

$$\int_C -\tfrac{1}{4}y^2\,dx + \tfrac{1}{2}xy\,dy$$

where C is the boundary of the region R, oriented counterclockwise.

6. (25 points)
(a) Consider the disk R consisting of points (x, y) with $(x - \frac{1}{2})^2 + y^2 \leq \frac{1}{4}$. Write an equation for the boundary in polar coordinates in the form $r = f(\theta)$, $\alpha \leq \theta \leq \beta$.
Hint: $(x - \frac{1}{2})^2 + y^2 = \frac{1}{4} \iff x^2 + y^2 - x = 0$.
(b) The region D consists of the points (x, y, z) where $0 \leq z \leq 1 - x^2 - y^2$ and $(x, y) \in R$. Find the volume of D.

7. (25 points) Consider the vector field

$$\mathbf{F}(x, y, z) = e^x \cos z\,\mathbf{i} + y\,\mathbf{j} - e^x \sin z\,\mathbf{k}.$$

(a.) Is $\mathbf{F}$ a conservative vector field? Justify your answer.
(b.) Evaluate

$$\int_C \mathbf{F} \cdot d\mathbf{r},$$

where the curve C is parametrized by $\mathbf{r}(t) = e^{t^2(t^2-1)}\,\mathbf{i} + t\,\mathbf{j} + te^{t^{17}-1}\,\mathbf{k}$, for $0 \leq t \leq 1$.

8. (25 points) Let D denote the region consisting of the points (x, y, z) with $0 \leq x \leq 1$, $0 \leq y \leq 1$, $0 \leq z \leq x$. Let Σ denote the boundary of D. Consider the vector field

$$\mathbf{F}(x, y, z) = (e^x + e^{yz})\mathbf{i} + (y^2 - z^2)\mathbf{j} + e^z\mathbf{k}.$$

Use the divergence theorem to evaluate the following integral

$$\iint_\Sigma \mathbf{F} \cdot \mathbf{n}\,dS,$$

where $\mathbf{n}$ is the normal to Σ that is directed outward.