Math 241 Chapter 15 Integral Study Guide

Important 1: Curves are always parametrized by $\bar{r}(t) = x(t)\,\hat{\imath} + y(t)\,\hat{\jmath} + z(t)\,\hat{k}$ for $a \le t \le b$ Note that some components might be 0 and the $\hat{k}$ component will definitely be 0 in 2D.
Important 2: Surfaces are always parametrized by $\bar{r}(u,v) = x(u,v)\,\hat{\imath} + y(u,v)\,\hat{\jmath} + z(u,v)\,\hat{k}$ for u, v restricted by R. Note that often it's not u, v but x, y or z, θ or ...
Important 3: Keep in mind that $\int_C 1\ ds$ is length of C, $\iint_\Sigma 1\ dS$ is surface area of Σ, $\iint_R 1\ dA$ is area of R and $\iiint_D 1\ dV$ is volume of D.

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1. Line integral of a function. One choice:

$$\int_C f\ ds = \int_a^b f(x(t), y(t), z(t))||\bar{r}'(t)||\ dt$$

2. Line integral of a vector field. Lots of possibilities. This one has the most options.

   (a) Most basic. Could be so ugly as to be nonintegrable:

$$\int_C \bar{F} \cdot d\bar{r} = \int_a^b \bar{F}(x(t), y(t), z(t)) \cdot \bar{r}'(t)\ dt$$

   (b) If $\bar{F}$ is conservative and f is a potential function then:

$$\int_C \bar{F} \cdot d\bar{r} = f(\text{endpt of C}) - f(\text{startpt of C})$$

   (c) If $\bar{F}$ is conservative and C is closed then:

$$\int_C \bar{F} \cdot d\bar{r} = 0$$

   (d) If C is the edge of Σ with induced orientation then Stokes's Theorem gives:

$$\int_C \bar{F} \cdot d\bar{r} = \iint_\Sigma (\nabla \times \bar{F}) \cdot \bar{n}\ dS \quad \text{Then go to 4(a)}$$

   (e) Note that there is alternate notation for this:

$$\int_C M dx + N dy + P dz \quad \text{means} \quad \int_C (M\,\hat{\imath} + N\,\hat{\jmath} + P\,\hat{k}) \cdot d\bar{r}$$

   (f) If in 2D and C is the edge of R with the cows on the left then Green's Theorem gives:

$$\int_C (M\,\hat{\imath} + N\,\hat{\jmath}) \cdot d\bar{r} \ \text{ or } \int_C M dx + N dy = \iint_R N_x - M_y\ dA$$

3. Surface integral of a function. One choice:

$$\iint_\Sigma f\ dS = \iint_R f(x(u,v), y(u,v), z(u,v))||\bar{r}_u \times \bar{r}_v||\ dA$$

4. Surface integral of a vector field. Two possibilities.

   (a) Always unless the Divergence Theorem applies:

$$\iint_\Sigma \bar{F} \cdot \bar{n}\ dS = \pm \iint_R \bar{F}(x(u,v), y(u,v), z(u,v)) \cdot [\bar{r}_u \times \bar{r}_v]\ dA$$

   (b) If Σ is the boundary surface of a solid object D with outward orientation then we can use the Divergence Theorem (Gauss's Theorem) giving:

$$\iint_\Sigma \bar{F} \cdot \bar{n}\ dS = \iiint_D \nabla \cdot \bar{F}\ dV$$