

Math 241 Exam 3 Sample 3

Directions: Do not simplify unless indicated. No calculators are permitted. Show all work as appropriate for the methods taught in this course. Partial credit will be given for any work, words or ideas which are relevant to the problem.

Label: The word “label” means basically label the functions in the picture and any important variable values. Essentially only those things which are important to the problem.

Please put problem 1 on answer sheet 1

1. (a) Parametrize the portion of $z = 9 - x^2 - y^2$ above the xy -plane. [5 pts]
- (b) Draw the surface $r(x, y) = x \hat{i} + y \hat{j} + (4 - y^2) \hat{k}$ for $0 \leq x \leq 3$ and $0 \leq y \leq 2$. [5 pts]
- (c) Evaluate the double integral $\int_0^1 \int_0^{\sqrt{1-x^2}} \sin(x^2 + y^2) dy dx$ by reparametrizing the region as polar. [10 pts]

Please put problem 2 on answer sheet 2

2. (a) Let R be the region between the graphs of $x = 0$, $y = x$ and $y = \frac{1}{2}x + 4$. Draw and label a picture of R . Set up an iterated integral in rectangular coordinates treating R as vertically simple for $\iint_R y dA$. **Do Not Evaluate.** [10 pts]
- (b) Let R be the region inside $r = 2 \cos \theta$ and outside $r = 1$. Draw and label a picture of R . Set up an iterated integral in polar coordinates for $\iint_R \frac{x}{y} dA$. **Do Not Evaluate.** [10 pts]

Please put problem 3 on answer sheet 3

3. (a) Draw, label and shade the region R parametrized by $\int_0^\pi \int_0^{\sin y} \dots dx dy$. [7 pts]
- (b) Draw, label and shade the region R parametrized by $\int_{-\pi/4}^{\pi/4} \int_1^{2 \sec \theta} \dots dr d\theta$. Hint: $r = 2 \sec \theta$ is $r \cos \theta = 2$. [7 pts]
- (c) Convert $z + \sqrt{x^2 + y^2} = 4$ to cylindrical coordinates and rewrite as $z = \dots$ [6 pts]

Please put problem 4 on answer sheet 4

4. (a) Let D be the solid object above the xy -plane, below the parabolic sheet $z = 9 - x^2$ and inside the cylinder $r = 1$. Draw and label separate pictures of D and R and then set up the iterated triple integral in rectangular coordinates for the mass of D if $\delta(x, y, z)$ equals the height of the point above the xy -plane. **Do Not Evaluate.** [12 pts]
- (b) Let D be the solid object between the cones $z = \sqrt{3x^2 + 3y^2}$ and $z = \sqrt{\frac{1}{3}x^2 + \frac{1}{3}y^2}$ and inside the sphere $x^2 + y^2 + z^2 = 9$ and in the first octant. Draw and label a picture of D and then set up the iterated integral in spherical coordinates for the volume of D . **Do Not Evaluate.** [8 pts]

Please put problem 5 on answer sheet 5

5. Perform a change of variables intended to evaluate $\iint_R x dA$, where R is the region inside the ellipse $4x^2 + 9y^2 = 36$. Draw and label pictures of R , your new region S and keep going until you have a double integral in polar coordinates. **Do Not Evaluate.** [20 pts]

The End!