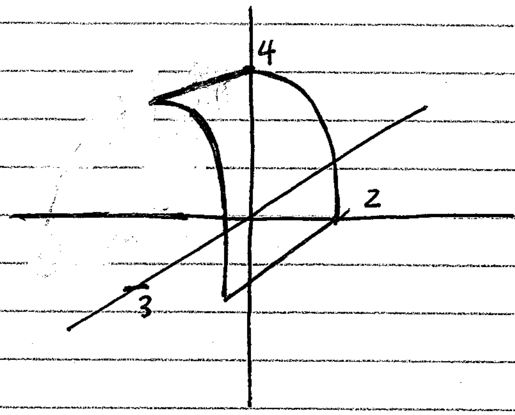
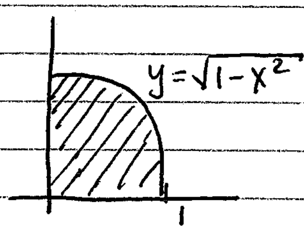
1(a) We do $\bar{r}(r,\theta) = r\cos\theta\,\hat{i} + r\sin\theta\,\hat{j} + (9-r^2)\,\hat{k}$
for $0 \le \theta \le 2\pi$ and $0 \le r \le 3$

(b) Since $z = 4-y^2$ this is a part of a parabolic sheet:



(c) The param $\int_0^1 \int_0^{\sqrt{1-x^2}} \ldots\, dy\,dx$ is VS and looks like



Reparam as polar to get

$$\int_0^{\pi/2} \int_0^1 \sin(r^2)\, r\, dr\, d\theta$$

$$= \int_0^{\pi/2} -\frac{1}{2}\cos(r^2)\Big|_0^1\, d\theta$$

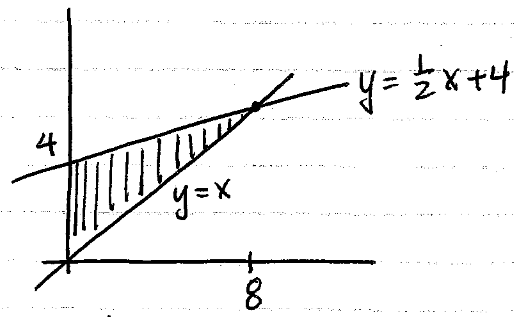
$$= \int_0^{\pi/2} -\frac{1}{2}\cos(1) + \frac{1}{2}\cos(0)\, d\theta$$

$$= \int_0^{\pi/2} \frac{1}{2} - \frac{1}{2}\cos(1)\, d\theta$$

$$= \frac{1}{2}\theta - \frac{1}{2}\theta\cos(1)\Big|_0^{\pi/2}$$

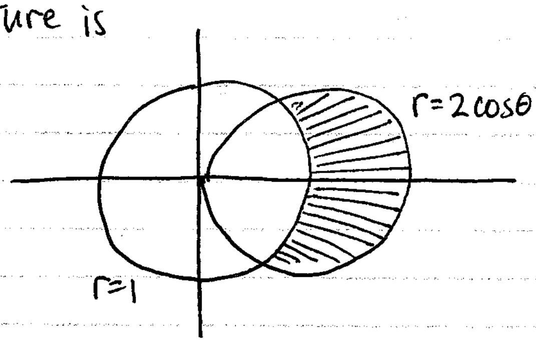
$$= \left[\frac{1}{2}\cdot\frac{\pi}{2} - \frac{1}{2}\frac{\pi}{2}\cos(1)\right] - \left[0-0\right]$$

2 (a) The two diagonal lines meet when $y = x$ meets $y = \frac{1}{2}x + 4$ which is at $\frac{1}{2}x + 4 = x$ or $x = 8$



the integral is $\int_0^8 \int_x^{\frac{1}{2}x+4} y \, dy \, dx$

(b) The picture is

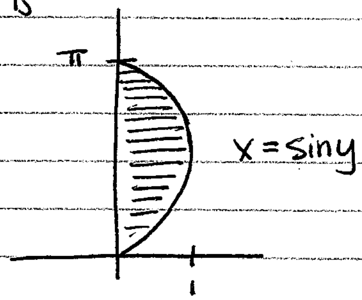


These circles meet when $r=1$ meets $r = 2\cos\theta$:

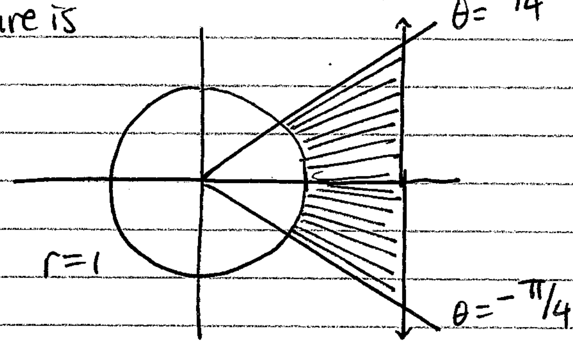
$$2\cos\theta = 1$$
$$\cos\theta = \frac{1}{2}$$
$$\theta = \pm \pi/3$$

So the integral is $\int_{-\pi/3}^{\pi/3} \int_1^{2\cos\theta} \frac{r\cos\theta}{r\sin\theta} \, r \, dr \, d\theta$

3(a) The picture is



(b) The picture is



$r = 2\sec\theta$

$r\cos\theta = 2$

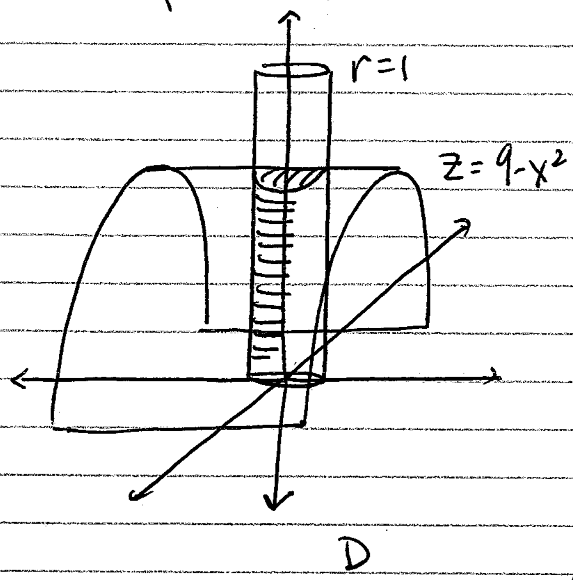
$x = 2$

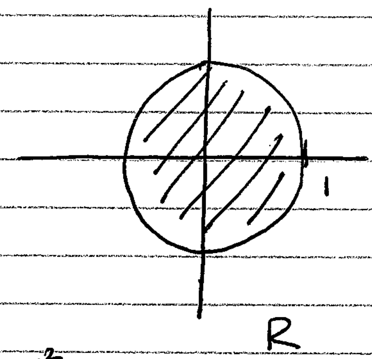
(c) We have $z = 4 - \sqrt{x^2+y^2}$

$z = 4 - \sqrt{r^2}$

$z = 4 - r$

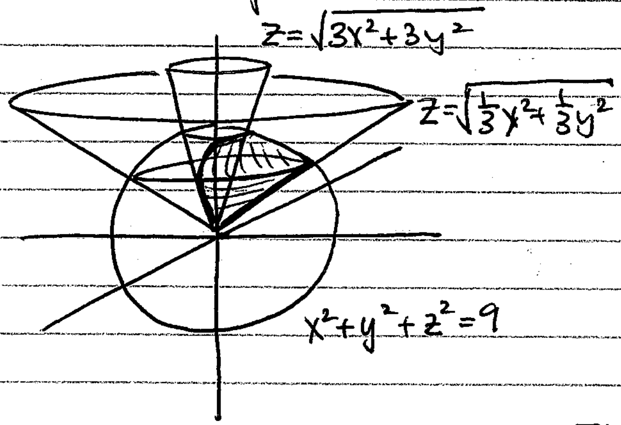
4(a) The pictures of D and R are

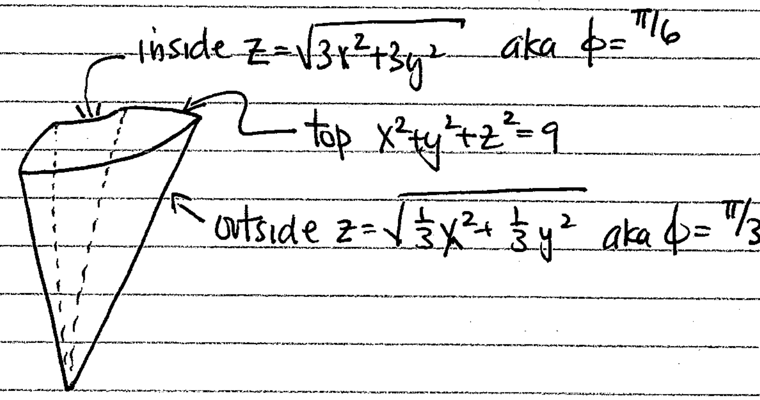




The mass is $\iiint_D \delta\, dV = \int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_0^{9-x^2} z\, dz\, dy\, dx$

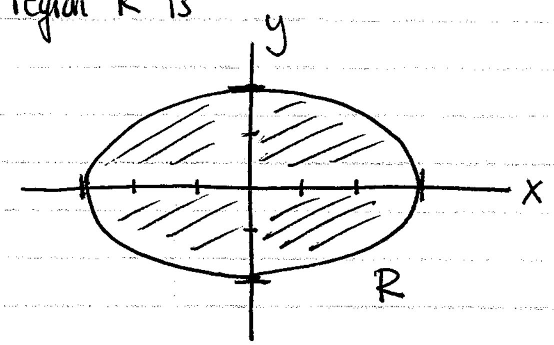
(b) Here are two pictures





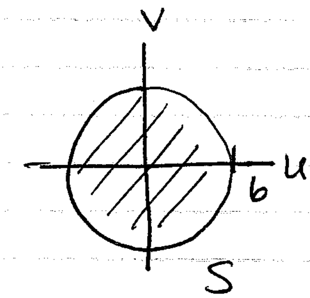
The volume is $\iiint_D 1\, dV = \int_0^{\pi/2}\int_{\pi/6}^{\pi/3}\int_0^{3} \rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$

5. The region R is



we rewrite $4x^2+9y^2=36$

$$(2x)^2+(3y)^2=36$$



and sub $u=2x$ and $v=3y$ so we get $u^2+v^2=36$

Then $x=\frac{1}{2}u$ and $y=\frac{1}{3}v$

and so $J=\begin{vmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{3} \end{vmatrix} = \frac{1}{6}$

so

$$\iint_R x\,dA = \iint_S \frac{1}{2}u \left|\frac{1}{6}\right| dA$$

then we param S in polar

$u=r\cos\theta$

$v=r\sin\theta$

$$=\int_0^{2\pi}\int_0^6 \frac{1}{12} r\cos\theta \cdot r\,dr\,d\theta$$