SAMPLE 1 SOLNS

1. $f(x,y) = 3x^2 + 6xy - 2y^3$

(a) $\bar{u} = \dfrac{3\hat{i} + 1\hat{j}}{\sqrt{10}} = \dfrac{3}{\sqrt{10}}\hat{i} + \dfrac{1}{\sqrt{10}}\hat{j}$

$f_x = 6x + 6y$ and $f_y = 6x - 6y^2$

So $D_{\bar{u}} f = \frac{3}{\sqrt{10}}(6x+6y) + \frac{1}{\sqrt{10}}(6x - 6y^2)$

(b) We set $f_x = 0$ and $f_y = 0$

$6x + 6y = 0$ tells us $y = -x$

$6x - 6y^2 = 0$ becomes $6x - 6x^2 = 0$

$6x(1-x) = 0$

$x = 0$ OR $x = 1$

If $x = 0$ then $y = -x = 0$ $(0,0)$

If $x = 1$ then $y = -x = -1$ $(1,-1)$

(c) $f_{xx} = 6$ $f_{yy} = -12y$ $f_{xy} = 6$

$D(x,y) = (6)(-12y) - (6)^2$

At $(0,0)$ $D(0,0) = -$ So saddle point

At $(1,-1)$ $D(1,-1) = (6)(12) - (6)^2 = +$

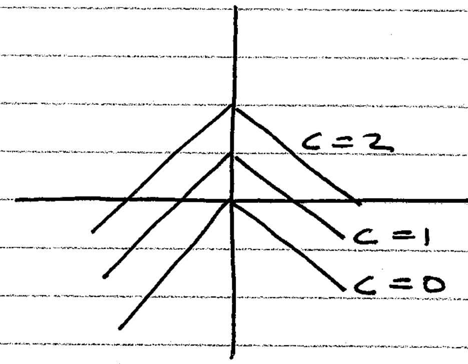
$f_{xx}(1,-1) = 6 = +$ So rel. min.

2. (a) $f(x,y) = |x| + y$

$c=0$: $|x| + y = 0$ $\quad y = -|x|$

$c=1$: $|x| + y = 1$ $\quad y = 1 - |x|$

$c=2$: $|x| + y = 2$ $\quad y = 2 - |x|$



(b) The line perp to the surface has direction vector perp to the surface so we need the surface as the level surface.
so let:

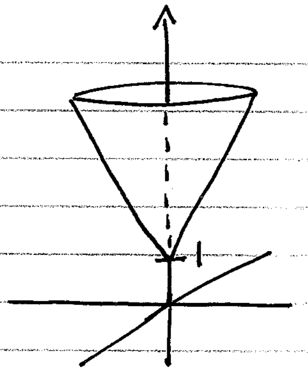
$$f(x,y) = x^2y + y^3$$
$$z = x^2y + y^3$$
$$x^2y + y^3 - z = 0$$
$$g(x,y,z) = x^2y + y^3 - z$$

then $\nabla g = (2xy)\hat{\imath} + (x^2 + 3y^2)\hat{\jmath} - 1\hat{k}$

at $(1,2,f(1,2)) = (1,2,10)$: $\nabla g(1,2,10) = 4\hat{\imath} + 13\hat{\jmath} - 1\hat{k}$

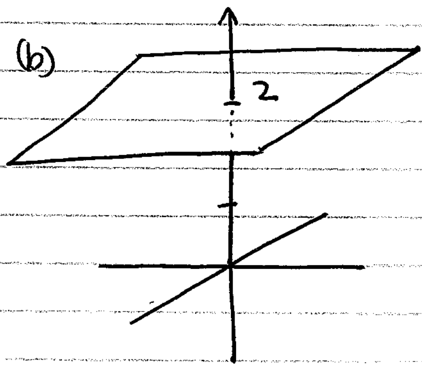
so
$$x = 1 + 4t$$
$$y = 2 + 13t$$
$$z = 10 - 1t$$

3. (a)



cone

(b)



plane

(c) $y^2+z^2=9$

(d) $z=7-x^2-y^2$

4. Skip!

5. The constraint is $g(x,y) = x^2+y^2=4$ so

$f_x = y+2 \qquad g_x = 2x$
$f_y = x \qquad g_y = 2y$

So we solve
$$y+2 = \lambda \cdot 2x \quad (a)$$
$$x = \lambda \cdot 2y \quad (b)$$
$$x^2+y^2 = 4 \quad (c)$$

(a) tells us $\lambda = \frac{y+2}{2x}$ unless $x=0$ so we check this case.
If $x=0$ then (b) says either $\lambda=0$ or $y=0$.
$\lambda=0$ into (a) says $y=-2$ which is okay in (c) so $(0,-2)$ is valid.
$y=0$ with $x=0$ is not okay in (c)

(b) tells us $\lambda = \frac{x}{2y}$ unless $y=0$ but then (b) says $x=0$ and we know these are not okay in (c)

So we have $\frac{x}{2y} = \frac{y+2}{2x}$ or $x^2 = y^2+2y$

Into (c) to get
$$y^2+2y+y^2=4$$
$$y^2+y-2=0$$
$$(y+2)(y-1)=0$$
$$y=-2 \text{ or } y=1$$

If $y=-2$ then (c) says $x=0$ which we have already.
if $y=1$ then (c) says $x=\pm\sqrt{3}$ giving $(\pm\sqrt{3},1)$

$f(0,-2) = 0$
$f(\sqrt{3},1) = 3\sqrt{3}$ Maximum
$f(-\sqrt{3},1) = -3\sqrt{3}$ Minimum