Math 241 Exam 3 Sample 2

Directions: Do not simplify or evaluated unless indicated. No calculators are permitted. Show all work as appropriate for the methods taught in this course. Partial credit will be given for any work, words or ideas which are relevant to the problem.

Please put problem 1 on answer sheet 1

1. (a) Write down the iterated double integral for $\iint\limits_R x\ dA$, where R is the region in the plane [10 pts] between $y = 4 - (x-1)^2$ and $y = 3 - x$, treating R as vertically simple.

   (b) Write down the iterated double integral(s) in polar coordinates necessary to calculate [10 pts] $\iint\limits_R \frac{x}{y+2}\ dA$, where R is the region inside $(x-1)^2 + y^2 = 1$ and to the left of $x = 1$.

Please put problem 2 on answer sheet 2

2. Let R be the region in the plane bounded by the lines $x = 1$, $x = 4$, $y = 2x+1$ and $y = 2x+5$. [20 pts] Draw a picture of R and label all the boundary curves. Use an appropriate change of variables to rewrite $\iint\limits_R \frac{y}{x}\ dA$ as an iterated integral. Show all work, including the Jacobian of your transformation, your $x =$ and $y =$, and your new region S.

Please put problem 3 on answer sheet 3

3. (a) Convert $x^2 + y^2 + (z-1)^2 = 1$ to spherical and rewrite as $\rho = $ .... Draw a picture. [5 pts]

   (b) Convert $x = y$ to spherical and rewrite as $\theta = $ .... Draw a picture. [5 pts]

   (c) Parametrize the rectangle with vertices $(3, 0, 0)$, $(3, 0, 2)$, $(0, 3, 0)$ and $(0, 3, 2)$. [5 pts]

   (d) Draw the surface $r(\theta, z) = 2\cos\theta\ \hat{\imath} + 2\sin\theta\ \hat{\jmath} + z\ \hat{k}$ for $0 \le \theta \le \pi$ and $0 \le z \le 4$. [5 pts]

Please put problem 4 on answer sheet 4

4. (a) Let D be the solid object inside the cylinder $r = 1$, outside the cylinder $r = \cos\theta$, in [10 pts] the first octant, and below $z = 3 - \sqrt{x^2 + y^2}$. Draw a picture of D. Write an iterated integral in cylindrical coordinates for the volume of D.

   (b) Let D be the solid object above the xy-plane, inside the cylinder $y^2 + z^2 = 4$ and between [10 pts] $x = -1$ and the parabolic sheet $x = 9 - y^2$. Draw a picture of D. Write an iterated integral in rectangular coordinates for $\iiint\limits_D xy\ dV$.

Please put problem 5 on answer sheet 5

5. Let D be the solid object between the spheres of radius 2 and 3, and having $y \le 0$ and $x \le 0$. [20 pts] Set up and evaluate the iterated triple integral

$$\iiint\limits_D \frac{1}{\sqrt{(x^2 + y^2)(x^2 + y^2 + z^2)}}\ dV$$

in spherical coordinates. Draw a picture of D. If you are careful with your work the evaluation of the integral should be straightforward.

The End!