MATH 310: Exam 1 **Monday 7/23/2012**

1. What is the cardinality of $\{\mathbb{Z}, \{1,2\}, 2, \mathbb{Q}, \emptyset\}$? [3 pts]
2. Write the elements in $\mathcal{P}(\mathcal{P}(\{1\}))$. [7 pts]
3. Let $S = \{0, 3, 6, 9, 12\}$. Describe the set $\{1, 2, 3, 4\}$ in the form $\{f(x) \mid x \in S \text{ and } p(x)\}$ for some functions $f(x)$ and $p(x)$. [10 pts]
4. Fill in the following truth table only for the possibilities given. [15 pts]

P	Q	R	$P \wedge Q$	$(P \wedge Q) \to R$	$R \to (P \wedge Q)$
T	T	T			
T	F	T			
F	T	F			
F	F	F			

5. Consider the open sentences over the domain $\mathcal{P}(\{1,2,3,4\})$ [10 pts]

$$P(A) : A \cap \{1\} \neq \emptyset \text{ and } Q(A) : |A| \leq 3$$

   Explicitly list all sets $A \in \mathcal{P}(\{1,2,3,4\})$ for which $P(A) \wedge Q(A)$ is true.

6. Consider the open sentences over the domain $\mathbb{R}$ [10 pts]

$$P(x) : x - 1 \geq 0 \text{ and } Q(x) : x^2 + 3x \leq 0$$

   Find all $x \in \mathbb{R}$ such that $P(x) \to Q(x)$ is true. List as intervals.

7. Determine if the following are true or false, with justification.
   (a) $\exists x \in \{1, 2, 3\}, 3|5x - 1$ [5 pts]
   (b) $\forall x \in \mathbb{N}, 2x + 1$ is prime. [5 pts]
8. Let $a \in \mathbb{Z}$. Prove that a is even iff a^2 is even. [20 pts]
9. Let $n \in \mathbb{Z}$. Prove that if $n \not\equiv 0 \pmod 3$ then $n^2 \equiv 1 \pmod 3$. [15 pts]

Extra Credit: In class we discussed negating $\exists$ and $\forall$ but not $\exists!$. Figure out how to rewrite the following: $\sim [\exists! x, P(x)]$ using any symbols except $\exists!$. [$\leq$ 5 pts]