MATH 310: Exam 2 **Monday 8/6/2012**

1. Indicate what you would assume when proving each of the following by contradiction:
   (a) $\forall x, (P(x) \vee Q(x)) \to (R(x) \wedge (\sim S(x)))$ [3 pts]
   (b) $P \to (Q \to R)$ [2 pts]
2. Suppose the distinct equivalence classes for some equivalence relation on $A = \{1, 2, 3, 4, 5, 6, 7\}$ are $\{1, 2, 5\}$, $\{3, 6\}$, $\{4, 7\}$.
   (a) Is $2R6$ true or false? [1 pts]
   (b) List the elements in $\{x \in A \mid x \not R 1\}$. [2 pts]
   (c) List the elements in $[3]$. [2 pts]
3. Prove that $3^n > 5n^2$ for $n \in \mathbb{N}$ with $n \geq 4$. [10 pts]
4. Prove that the function $f : (\mathbb{R} - \{2\}) \to (\mathbb{R} - \{3\})$ defined by $f(x) = \frac{3x+6}{x-2}$ is surjective. [10 pts]
5. Define the relation R on $\mathbb{Z}$ by aRb if $3|(a - 4b)$. Prove that R is an equivalence relation. [15 pts]
6. By passing to equivalence classes modulo a judicious choice of n prove that $x^5+7x^2+4x+2 = 0$ has no integer solutions. [10 pts]
7. Prove there is a real solution but no rational solution to $x^5 - x + 1 = 0$. [15 pts]
   Important: For the second part you may assume basic facts about the products and sums of odd and even integers.
8. Prove that the sum of the squares of two odd integers cannot be a perfect square. [10 pts]
9. We proved on the homework that R on $\mathbb{Z}$ defined by aRb if $a^2 + b^2$ is even is an equivalence relation. Rigorously prove that $[1] = \{..., -3, -1, 1, 3, ...\}$. [10 pts]
   Important: You may assume here that $a \in \mathbb{Z}$ is odd iff a^2 is odd.
10. Prove that if $n \in \mathbb{Z}$ with $n \not\equiv 1 \pmod 3$ then $3|(2n^3 + 2n^2)$. [10 pts]