**MATH 310: Exam 3** **Friday 8/17/2012**

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1. Prove that for $n \in \mathbb{N}$ we have $1 + 3 + 3^2 + ... + 3^n < 3^{n+1}$. [4 pts]
2. Find the coefficient of $x^2y^5$ in the expression $(3x - 2y)^7$. [6 pts]
3. Prove that for $n \in \mathbb{N}$ and $b \in \mathbb{R}^+$ we have $(2 + b)^n \geq b^n + 2nb^{n-1}$. [5 pts]
4. Prove that $f : (\mathbb{R} - \{0\}) \to (\mathbb{R} - \{2\})$ given by $f(x) = \frac{2x+1}{x}$ is invertible and find $f^{-1}(y)$. [10 pts]
5. Give an explicit example of a non-invertible surjective function $f : \mathbb{R} \to \mathbb{R}$. Prove non-invertibility but do not prove surjectivity. [10 pts]
6. Prove that $\left\{\frac{n^2}{2n^2+3}\right\}$ does not converge to 1. [10 pts]
7. (a) Prove that if $\{a_n\}$ and $\{b_n\}$ both converge to some $c$ then for any $\epsilon$ there is some $N \in \mathbb{N}$ such that for $n > N$ we have $|a_n - b_n| < \epsilon$. [10 pts]

   (b) Provide a counterexample to prove that the converse of (a) is false. You do not need to prove your counterexample works. [5 pts]
8. Define [10 pts]

$$f(x) = \begin{cases} x + 1 & \text{if } x < 2 \\ 7 - x^2 & \text{if } x \geq 2 \end{cases}$$

   Prove that $f'(2)$ does not exist.

9. Prove that $f(x) = 2x^2 - 3x + 1$ is continuous at $x = 2$. [15 pts]
10. Find a bijection between $\mathbb{Q}^+$ and $\mathbb{N} \times \mathbb{N}$. You need not prove it's a bijection. [15 pts]