MATH 310: Groupwork 10 Solutions **Friday 7/20/2012**

1. Prove if $x, y \in \mathbb{R}$ are positive then $\sqrt{x+y} \neq \sqrt{x} + \sqrt{y}$.
Proof: Assume by way of contradiction that there exist $x \in \mathbb{R}$ which are positive and $\sqrt{x+y} = \sqrt{x} + \sqrt{y}$. Squaring both sides leads to $x + y = x + 2\sqrt{xy} + y$ which reduces to $2\sqrt{xy} = 0$ or $xy = 0$ which is impossible.
$\ddot\smile$

2. Prove that there is no positive integer n satisfying $2n < n^2 < 3n$.
Proof: Assume by way of contradiction that there is such an n. Then if we have $2n < n^2 < 3n$ we can divide by n to get $2 < n < 3$ which is impossible for an integer.
$\ddot\smile$

3. Two of the following are false and one is true. Prove the true one by contradiction and provide counterexamples for the false ones.
   (a) If $n^2 + 3n$ is even then n is odd.
   Counterexample: If $n = 0$ then $n^2 + 3n = 0$ is even but n is even.
   (b) If $a \geq 2$ and b are integers then $a \nmid b$ or $a \nmid (b+1)$.
   Proof: Assume that there exists $a \geq 2$ and b, both integers, with $a \mid b$ and $a \mid b+1$. Then the first yields $am = b$ and the second yields $an = b+1$ for $m, n \in \mathbb{Z}$. Then
$$\begin{aligned} an &= b + 1 \\ an &= am + 1 \\ an - am &= 1 \\ a(n-m) &= 1 \end{aligned}$$
   From whence it follows (since everyone's an integer) that either $a = \pm 1$, a contradiction.
   $\ddot\smile$
   (c) If $A \not\subseteq B$ then $A \cap B = \emptyset$.
   Counterexample: If $A = \{1, 2\}$ and $B = \{2, 3\}$ then clearly $A \not\subseteq B$ but $A \cap B = \{2\} \neq \emptyset$.