

1. For each of the following indicate symbolically what you would assume for each proof method: Direct, by contrapositive and by contradiction.

(a) $\forall x, (P(x) \wedge Q(x)) \rightarrow (R(x) \vee S(x))$

Direct:

Contrapositive:

Contradiction:

(b) $\forall x, \exists y, P(x, y) \rightarrow (Q(x) \wedge R(y))$

Direct:

Contrapositive:

Contradiction:

2. Prove by contradiction that $\sqrt{2} + \sqrt{3}$ is irrational.

3. Prove that $\exists x, y \in \mathbb{Z}, (10x - 8y = 4)$ has a solution.

4. Prove that there are infinitely many solutions to the above equation.

5. Prove that the equation $x^3 - 9x + 5 = 0$ has two real solutions.