1. Use induction to prove that $1(2)+2(3)+3(4)+...+n(n+1)=\frac{n(n+1)(n+2)}{3}$ for every positive integer n.

   Proof: We have:
   Base Case: When $n=1$ we check $1(2)=\frac{1(1+1)(1+2)}{3}$ which is true.
   Inductive Step: We assume $1(2)+2(3)+3(4)+...+n(n+1)=\frac{n(n+1)(n+2)}{3}$ and we wish to show $1(2)+2(3)+3(4)+...+n(n+1)+(n+1)(n+2)=\frac{(n+1)(n+2)(n+3)}{3}$. Observe:

$$\begin{aligned}
1(2)+2(3)+3(4)+...+n(n+1)+(n+1)(n+2) &= \frac{n(n+1)(n+2)}{3}+(n+1)(n+2) \quad \text{By IH}\\
&= \frac{n(n+1)(n+2)+3(n+1)(n+2)}{3}\\
&= \frac{(n+1)(n+2)(n+3)}{3}
\end{aligned}$$

   as desired. $\mathcal{QED}$

2. Use induction to prove that $4|(5^n-1)$ for every nonnegative integer n.

   Proof: We have:
   Base Case: When $n=0$ we check $4|(5^0-1)$ which is true.
   Inductive Step: We assume $4|(5^n-1)$ and we wish to show $4|(5^{n+1}-1)$. Observe that our assumption can be rewritten as $4x=5^n-1$ for $x\in\mathbb{Z}$ and then

$$5^{n+1}-1=5\cdot 5^n-1=5(4x+1)-1=20x+4=4(5x+1)$$

   and so $4|(5^{n+1}-1)$ as desired. $\mathcal{QED}$

3. Prove that $2^n>n^3$ for every integer $n\geq 10$.

   Proof: We have:
   Base Case: When $n=10$ we check $2^{10}>10^3$ which is true.
   Inductive Step: We assume $2^n>n^3$ and we wish to show $2^{n+1}>(n+1)^3$. Observe that

$$2^{n+1}-(n+1)^3>2\cdot 2^n-(n+1)^3>2n^3-(n+1)^3=n^3-3n^2-3n-1=n(n(n-3)-3)-1$$

   and then since $n\geq 10$ we know $n-3\geq 7$ and so $n(n-3)-3\geq 67$ and so $n(n(n-3)-3)-1\geq 669$ so that $2^{n+1}-(n+1)^3\geq 669>0$ and we have $2^{n+1}>(n+1)^3$. $\mathcal{QED}$