MATH 310: Groupwork 14 Semi-Solutions **Friday 7/27/2012**

1. Some of the following are true and some are not. First find counterexamples for the false ones and check with me that you haven't missed any. Then provide proofs on the reverse side for as many of the true ones as you can. One is an unsolved problem in mathematics.

   (a) Let $n \in \mathbb{Z}$. If $4n + 7$ is odd then n is even.
   Verdict: False. Counterexample when $n = 1$.

   (b) Every even integer is the sum of two odd integers.
   Verdict: True. Prove directly.

   (c) Every odd integer greater than 2 which isn't prime is the sum of two primes.
   Verdict: False. Counterexample is 27.

   (d) Every even integer greater than 2 is the sum of two primes.
   Verdict: Unsolved problem, the Goldbach Conjecture (that it's true).

   (e) For every two sets A and B, $(A \cup B) - B = A$.
   Verdict: False. Counterexample is $A = \{1, 2\}$, $B = \{2, 3\}$.

   (f) There is a real solution to $x^4 + x^2 + 1 = 0$.
   Verdict: False.

   (g) For every integer $n \geq 5$ we have $2^n > n^2$.
   Verdict: True. Prove by induction.

   (h) If $x, y \in \mathbb{R}$ and $x^2 < y^2$ then $x < y$.
   Verdict: False. Counterexample is $x = 2$ and $y = -3$.

   (i) Let A and B be sets. If $A - B = B - A$ then $A - B = \emptyset$.
   Verdict: True. Prove directly.

   (j) For every $a \in \mathbb{R}$ with $a > 0$ there is a rational number r with $0 < r < a$.
   Verdict: True. Prove directly.

   (k) For every two sets A and B we have $\mathcal{P}(A \cup B) = \mathcal{P}(A) \cup \mathcal{P}(B)$.
   Verdict: False. Counterexample is $A = \{1\}$ and $B = \{2\}$.

   (l) For every positive n, the integer $n^2 - n + 11$ is prime.
   Verdict: False. Counterexample is $n = 11$.

   (m) There is a real solution to $x^4 - 2x^3 + x - 2 = 0$.
   Verdict: True. Find either explicitly or implicitly.

   (n) For every integer $n \geq 1$ we have $n! \leq 2^n$.
   Verdict: False. Counterexample is $n = 4$.

   (o) Let $n \in \mathbb{Z}$. If $7n - 1$ is odd then n is even.
   Verdict: True. Prove by contrapositive.