1. Define $A = \{1, 2, 3\}$ and $B = \{x, y, z, w\}$ and define the relation R from A to B by

$$R = \{(1, y), (3, x), (3, z), (2, w), (2, y), (3, y)(1, x)\}$$

   (a) Is it true that $3Rx$?
   Answer: Yes because $(3, x) \in R$.

   (b) Is it true that $2Rx$?
   Answer: No because $(2, x) \notin R$.

   (c) List the elements in $\{n \in A \mid nRx\}$
   Answer: $\{1, 3\}$

   (d) List the elements in $\{\alpha \in B \mid 3R\alpha\}$
   Answer: $\{x, y, z\}$

2. Define $A = \{1, 2, 3, 4, 5, 6\}$. Suppose I start defining the relation $R = \{(1, 3), (3, 5), (2, 6)\}$. Add as many elements as necessary to R (but no more than necessary) to make sure that the relation is reflexive, symmetric and transitive.
   Answer: At the end we have:

$$R = \{(1, 3), (3, 5), (2, 6), (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6), (3, 1), (5, 3), (2, 6), (1, 5), (5, 1)\}$$

3. Define $A = \{1, 2, 3, 4\}$. Give an example of a nonempty relation on A which is:

   (a) Symmetric and reflexive but not transitive.
   Answer: One answer is $\{(1, 1), (2, 2), (3, 3), (4, 4), (1, 2), (2, 1), (2, 3), (3, 2)\}$.

   (b) Transitive but neither symmetric nor reflexive.
   Answer: One answer is $\{(1, 2), (2, 3), (1, 3)\}$.

4. Define a relation R on $\mathbb{Z}$ by $R = \{(x, y) \mid |x - y| \geq 1\}$. Show that R is symmetric but is neither reflexive nor transitive.
   Answer:
   R is symmetric because if xRy then $|x - y| \geq 1$ and then $|y - x| = |x - y| \geq 1$.
   R is not reflexive since, for example, $1 \not R\, 1$ since $|1 - 1| \ngeq 1$.
   R is not transitive since, for example, $0R1$ and $1R0$ but $0 \not R\, 0$. In friendlier terms $|0 - 1| \geq 1$ and $|1 - 0| \geq 1$ but $|0 - 0| \ngeq 1$.

5. Define a relation R on $\mathbb{Z}$ by $R = \{(x, y) \mid 4|(x - y)\}$. Which properties (reflextive, symmetric, transitive) does R have? Provide proofs.
   Answer: Similar problem tomorrow, no answer given here.

6. Suppose A is the set of all students in this class and we define a relation R on A by sRt if students s and t have a birthday in the same month. Let x be you. Find the set $\{y \in A \mid xRy\}$.
   Answer: Not given!