1. Write down a multiplication table for the equivalence classes of the integers modulo 3.

   Solution: We have:

$\cdot$	0	1	2
0	0	0	0
1	0	1	2
2	0	2	1

2. By passing to equivalence classes modulo a judicious choice of n, solve the following problems.

   (a) Show that $x^5 + 7x^2 + 4x + 2 = 0$ has no integer solutions.

   Proof: Assume by way of contradiction that a solution exists Passing to equivalence classes with $n = 3$ we get:

$$\begin{aligned} x^5 + 7x^2 + 4x + 2 &= 0 \\ [x]^5 + [7][x]^2 + [4][x] + [2] &= [0] \\ [x]^5 + [x]^2 + [2] &= [0] \end{aligned}$$

   Now then there are only three possible equivalence classes with $n = 3$ so we test them all:

   $[x] = 0$: $[0]^5 + [0]^2 + [2] = [2] \neq [0]$
   $[x] = 1$: $[1]^5 + [1]^2 + [2] = [1] \neq [0]$
   $[x] = 2$: $[2]^5 + [2]^2 + [2] = [2] \neq [0]$

   We therefore have a contradiction and no integer solutions exist.

   $\mathcal{QED}$

   (b) Show that $46^3 + 23^5 \neq 6533678$.

   Proof: Assume they're equal and pass to equivalence classes modulo $n = 2$ to get $[0]^3 + [1]^5 = [0]$ which is false. Therefore they're not equal.

   $\mathcal{QED}$

   (c) Show that $15x - 12y = 100$ has no integer solutions.

   Proof: Assume by way of contradiction that a solution exists. Passing to equivalence classes modulo $n = 3$ yields $[15][x] - [12][y] = [100]$ or $[0] = [1]$, a contradiction. Therefore no integer solutions exists.

   $\mathcal{QED}$

   (d) Show that if $a \in \mathbb{Z}$ then $a^2 \not\equiv 2 \pmod 4$ and $a^2 \not\equiv 3 \pmod 4$.

   Solution: This was put on HW8 so I won't provide a solution here. A hint though is to proceed by contradiction and rewrite the equivalences as statements about equivalence classes with the screamingly obvious choice of n.