MATH 310: Groupwork 18 **Thursday 8/2/2012**

1. Prove that the function $f : [0, \infty) \to \mathbb{R}$ given by $f(x) = \frac{x}{x+1}$ is injective.

2. Prove that the function $f : \mathbb{Z} \to \left\{ \left(\frac{1}{2}\right)^a \mid a \in \mathbb{Z}, a \geq 0 \right\}$ given by $f(x) = \left(\frac{1}{2}\right)^{|2-x|}$ is surjective and sketch the range on the number line.

3. Prove that the function $f : (\mathbb{R} - \{0, 1\}) \to (\mathbb{R} - \{0\})$ given by $f(x) = \frac{1}{x(x-1)}$ is surjective.

   Note: This problem actually has an error as written that only one person caught, and it wasn't me. Once you set $\frac{1}{x(x-1)} = y$ and solve for x you get an expression that is undefined for $0 < y < 4$. In reality the function is only surjective if the codomain is $\mathbb{R} - (-4, 0]$. The reason this was hard to catch was that solving for x is pretty straightforward but it's easy to overlook that the expression you get is not always defined.