MATH 310: Groupwork 20 Solutions **Tuesday 8/7/2012**

1. Let $A = \{(a, b) \mid a, b \in \mathbb{N}, a \leq b\}$. Show A is countable by giving a coherent explicit listing (or explanation of a listing) of the elements.

   Solution: For each b we iterate a through $1, 2, ..., b$ and then increase b so our listing is:

   $$(1,1),(1,2),(2,2),(1,3),(2,3),(3,3),(1,4),...$$

2. Find a bijection $f : \mathbb{Z} \to (\mathbb{Z} - \{2\})$. You do not need to prove it is a bijection. You'll probably need a piecewise defined function.

   Solution: The easiest way is a simple piecewise function:

   $$f(n) = \begin{cases} n & \text{if } n \leq 1 \\ n+1 & \text{if } n \geq 2 \end{cases}$$

3. Suppose a hotel has a countably infinite number of rooms numbered $1, 2, 3, ...$ all of which are full. None of the guests can leave but they can be sent to other rooms.

   (a) One new guest arrives. Explain how the hotel can fit him in.

   Solution: For all n move the guest in room n to room $n + 1$, leaving room 1 empty. Put the new guest there.

   (b) Countably infinitely many new guests arrive. Explain how the hotel can fit them all in.

   Solution: For all n move the guest in room n to room $2n$, leaving the odd rooms empty. Put new guest k in room $2k - 1$.

   (c) Countably infinitely many groups each with countably infinitely many new guests arrive. Explain how the hotel can fit them all in.

   Solution: First for all n move the guest in room n room $2n$, leaving the odd rooms empty.

   Then arrange the new guests into an array as follows:

Group 1:	1	2	3	...
Group 2:	1	2	3	...
Group 3:	1	2	3	...
Group 4:	1	2	3	...

   Now follow the snake diagram from class to move these guests into rooms 1,3,5,7,...