1. Disprove by counterexample:

   (a) $|A| < |B|$ and $A$ countable implies $B$ countable.

   (b) $|A| < |B|$ and $B$ uncountable implies $A$ uncountable.

2. For each of the following sets write down a few elements which give a good idea of what sorts of elements each set has. Be creatively interesting.

   (a) $\mathbb{Z}$

   (b) $\mathcal{P}(\mathbb{Z})$

   (c) $\mathcal{P}(\mathcal{P}(\mathbb{Z}))$

   (d) $\mathcal{P}(\mathbb{R})$.

3. Prove that for $a, b, c, d \in \mathbb{R}$ with $a < b$ and $c < d$ we have $\Big|[a, b]\Big| = \Big|[c, d]\Big|$ by finding an explicit bijection between the sets.

4. Your hotel now has uncountably infinitely many rooms each numbered with a real number in $[0, 1]$ and all full. However uncountably infinitely many guests arrive, each with a number in $[0, 1]$. Explain how you can fit them all in.