1. Show that $\left\{\frac{4n-1}{2n+1}\right\}$ converges to 2.

   Idea: We need to show that for any ϵ we can find some N so that $n > N$ implies

$$\begin{aligned}\left|\frac{4n-1}{2n+1} - 2\right| &< \epsilon \\ \left|\frac{4n+1-2(2n+1)}{2n+1}\right| &< \epsilon \\ \frac{1}{2n+1} &< \epsilon \\ 2n+1 &> 1/\epsilon \\ n &> \frac{1/\epsilon - 1}{2}\end{aligned}$$

   Proof: Given ϵ we let $N = \lceil \frac{1/\epsilon - 1}{2} \rceil$. Then if $n > N$ we have

$$\begin{aligned}n &> \frac{1/\epsilon - 1}{2} \\ \frac{1}{2n+1} &< \epsilon \\ \left|\frac{4n+1-2(2n+1)}{2n+1}\right| &< \epsilon \\ \left|\frac{4n-1}{2n+1} - 2\right| &< \epsilon\end{aligned}$$

2. Show that $\left\{\frac{2}{3^n+1}\right\}$ converges to 0.

   Idea: We need to show that for any ϵ we can find some N so that $n > N$ implies

$$\begin{aligned}\left|\frac{2}{3^n+1} - 0\right| &< \epsilon \\ \frac{2}{3^n+1} &< \epsilon \\ 3^n + 1 &> 2/\epsilon \\ 3^n &> 2/\epsilon - 1 \\ n &> \log_3(2/\epsilon - 1)\end{aligned}$$

   Proof: Given ϵ we let $N = \lceil \log_3(2/\epsilon - 1) \rceil$. Then if $n > N$ we have

$$\begin{aligned}n &> \log_3(2/\epsilon - 1) \\ 3^n &> 2/\epsilon - 1 \\ 3^n + 1 &> 2/\epsilon \\ \frac{2}{3^n+1} &< \epsilon \\ \left|\frac{2}{3^n+1} - 0\right| &< \epsilon\end{aligned}$$

3. Show that $\left\{2 + \frac{5}{n} + \frac{1}{n^2} + \frac{10}{n^3}\right\}$ converges to 2.

   Idea: We need to show that for any ϵ we can find some N so that $n > N$ implies $\left|2 + \frac{5}{n} + \frac{1}{n^2} + \frac{10}{n^3} - 2\right| < \epsilon$. However observe that

$$\left|\frac{5}{n} + \frac{1}{n^2} + \frac{10}{n^3}\right| < \left|\frac{5}{n} + \frac{1}{n} + \frac{10}{n}\right| = \frac{16}{n}$$

   so it sufficies to get $\frac{16}{n} < \epsilon$ which we can get with $n > \frac{16}{\epsilon}$.

   Proof: Given ϵ we let $N = \lceil 16/\epsilon$. Then if $n > N$ we have

$$\left|\frac{5}{n} + \frac{1}{n^2} + \frac{10}{n^3}\right| < \left|\frac{5}{n} + \frac{1}{n} + \frac{10}{n}\right| = \frac{16}{n} < \epsilon$$

   as desired.