MATH 310: Groupwork 23 Solutions **Friday 8/10/2012**

1. Show that $\left\{2 + \frac{1}{n}\right\}$ does not converge to 1.

   Proof: We proceed by contradiction and assume that for any ϵ there is some N so that $n > N$ implies $\left|2 + \frac{1}{n} - 1\right| < \epsilon$. This is equivalent to $1 + \frac{1}{n} < \epsilon$. But for $\epsilon = 1$ this becomes $\frac{1}{n} < 0$ which is impossible for $n \in \mathbb{N}$.

   QED

2. Show that $\left\{(-1)^n \frac{n}{n+1}\right\}$ does not converge to -6.

   Proof: We proceed by contradiction and assume that for any ϵ there is some N so that $n > N$ implies $\left|(-1)^n \frac{n}{n+1} - (-6)\right| < \epsilon$. Since $n > N$ (for any N) includes both even and odd n values let's look at an even n, then we get $\frac{n}{n+1} + 6 < \epsilon$. But for $\epsilon = 1$ this becomes $\frac{n}{n+1} < -5$ or $n < -5n - 5$ or $n < -5/4$ which is impossible for $n \in \mathbb{N}$.

   QED

3. Show that $f(x) = 5x + 2$ is continuous at $x = -1$.

   Idea: Assume $\{x_n\}$ converges to -1. We claim that $\{f(x_n)\} = \{5x_n + 2\}$ converges to $f(-1) = -3$. In other words we claim that for any ϵ we can find some N so that $n > N$ implies $|5x_n + 2 - (-3)| < \epsilon$. But this latter inequality is $|x_n - (-1)| < \epsilon/5$.

   Proof: Assume $\{x_n\}$ converges to -1. Choose N so that $n > N$ implies $|x_n - (-1)| < \epsilon/5$. Then

   $$|5x_n + 2 - (-3)| = 5|x_n - (-1)| < \epsilon$$

   as desired.

   QED