

1. Define $f : \mathbb{R} \rightarrow \mathbb{R}$ by

$$f(x) = \begin{cases} -2x + 8 & \text{if } x < 3 \\ x - 1 & \text{if } x \geq 3 \end{cases}$$

Show that $f'(3)$ is undefined.

2. Define $f : \mathbb{R} \rightarrow \mathbb{R}$ by

$$f(x) = \begin{cases} x^2 & \text{if } x \in \mathbb{Q} \\ -x^2 & \text{if } x \in \mathbb{R} - \mathbb{Q} \end{cases}$$

Show that $f'(0) = 0$.

3. Define $f : \mathbb{R} \rightarrow \mathbb{R}$ by

$$f(x) = \begin{cases} x^2 & \text{if } x \leq 1 \\ 2x - 1 & \text{if } x > 1 \end{cases}$$

Show that f is continuous at $x = 1$ and $f'(1) = 2$.