1. Prove that for $a, b \in \mathbb{R}$ we have $|a| - |b| \leq |a + b|$.

   Proof: Observe that

   $$|a| = |(a+b) + (-b)| \leq |a+b| + |-b| = |a+b| + |b|$$

   and the result is immediate.

   $\mathcal{QED}$

2. Use the Geometric Sum Formula to find a simplified formula for

   $$\frac{1}{1+x^2} + \frac{1}{(1+x^2)^2} + ... + \frac{1}{(1+x^2)^n}$$

   Solution: We have

   $$\frac{1}{1+x^2} + \frac{1}{(1+x^2)^2} + ... + \frac{1}{(1+x^2)^n} = \frac{1 - (1/(1+x^2))^{n+1}}{1 - (1/(1+x^2))} - 1$$

3. Use a truncated version of the Binomial Theorem to show that for $n \in \mathbb{N}$ and $b \in \mathbb{R}^+$ we have $(1+b)^n \geq 1 + nb + \frac{n(n-1)}{2}b^2$.

   Proof: By the Binomial Theorem we have

   $$(1+b)^n = \sum_{k=0}^{n} \binom{n}{k} 1^{n-k} b^k \geq \binom{n}{0} + \binom{n}{1} b^1 + \binom{n}{2} b^2 = 1 + nb + \frac{n(n-1)}{2} b^2$$

   as desired.

   $\mathcal{QED}$

4. Find the coefficient of x^2y^5 in $(x+y)^7$.

   Solution: In the binomial expansion x^2y^5 appears when $k = 2$ and so that term is

$$\binom{7}{2} x^2y^5$$

   so the coefficient is $\binom{7}{2} = 21$.

5. For $n \in \mathbb{N}$ and $a, b \in \mathbb{R}^+$ use the Difference of Powers Formula to show that

$$a \leq b \text{ iff } a^n \leq b^n$$

   Proof: Equivalently we are trying to show that

$$a - b \leq 0 \text{ iff } a^n - b^n \leq 0.$$

   The difference of square formula states that

$$a^n - b^n = (a-b)\sum_{k=0}^{n-1} a^k b^{n-1-k}$$

   Since $a, b \in \mathbb{R}^+$ the summation part is positive and hence $a^n - b^n$ and $a - b$ have the same sign. Thus one is positive iff the other is.

   $\mathcal{QED}$