MATH 310: Homework 10 **Due Monday 8/13/2012**

1. Prove that $\Big|[0,1] \times [0,1]\Big| = \Big|[0,1]\Big|$ by explicitly finding a bijection between the sets and proving it is a bijection. [10 pts]
2. Let A and B be nonempty sets. Prove that $|A| \leq |A \times B|$. [5 pts]
3. Find an example of infinite sets A and B with $|A| < |A \times B|$. [5 pts]
4. Find bijections between the following sets. You can use pictures or explicit functions as long as your argument is clear. You do not need to prove bijectivity.
   (a) $\mathbb{Z}$ and $\mathbb{Q}^+$ [10 pts]
   (b) $\mathbb{Q}^+$ and $\mathbb{Q}$ [10 pts]
   (c) $\mathbb{N} \times \mathbb{N}$ and $\mathbb{Z} \times \mathbb{Z}$ [10 pts]
5. Show that $\left\{\frac{3n+1}{9n-1}\right\}$ converges to $\frac{1}{3}$. [10 pts]
6. Show that $\left\{(-1)^n n^2\right\}$ does not converge to 3. [10 pts]
7. Show that $\left\{\frac{4n^3+n^2+3n+1}{n^3}\right\}$ converges to 4. [10 pts]
8. Prove that if $\{a_n\}$ is a sequence which converges to a and also to b then $a = b$.
   Note: This may seem obvious but the point is to prove it rigorously from the definition. [20 pts]