**MATH 310: Homework 12** **Due Friday 8/17/2012**

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1. Define $f : \mathbb{R} \to \mathbb{R}$ by [10 pts]
$$f(x) = \begin{cases} x^2 - 2x + 2 & \text{if } x < 1 \\ -x^2 + 2x & \text{if } x \geq 1 \end{cases}$$
Show that $f'(1) = 0$.

2. Suppose $f : \mathbb{R} \to \mathbb{R}$ is a function such that $f'(3)$ is defined and $f(x) = 3x^2 - 2x$ for $x \in \mathbb{Q}$ and is unknown for other $x$. Find $f'(3)$. [10 pts]

3. Define $f : \mathbb{R} \to \mathbb{R}$ by [10 pts]
$$f(x) = \begin{cases} x - x^2 & \text{if } x \text{ is rational} \\ x + x^2 & \text{if } x \text{ is irrational} \end{cases}$$
Draw a believable graph of $f$ and show that $f'(0) = 1$.

4. Write out the binomial formula explicitly for $n = 2, 3, 4$. [5 pts]

5. Show that if $a \neq 0$ then [10 pts]
$$\frac{1}{a} = 1 + (1-a) + (1-a)^2 + \frac{(1-a)^3}{a}$$

6. Use the Triangle Inequality to prove $|a| - |b| \leq |a + b|$. [10 pts]

7. Prove that for $n \in \mathbb{N}$ and $a, b \in \mathbb{R}$ with $a \geq b \geq 0$ we have $a^n - b^n \geq nb^{n-1}(a - b)$. [15 pts]

8. Find counterexamples to each of the following: [10 pts]
   (a) $|a + b| = |a| + |b|$.
   (b) $|a - b| = |a| - |b|$.