MATH 310: Homework 5 **Due Thursday 7/26/2012**

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1. For each of the following indicate symoblically what you would assume for each proof method: Direct, by contrapositive and by contradiction.

   (a) $\forall x, P(x) \to Q(x)$

   (b) $P \to (Q \vee R)$

   (c) $\forall x, \exists y, P(x, y) \to (Q(x, y) \wedge R(x, y))$

   (d) $\forall x, (P(x) \vee Q(x)) \to R(x)$

   (e) $\forall x, P(x) \vee (Q(x) \to R(x))$

2. Prove that for $a, b, c \in \mathbb{Z}$ that if $a|(b + c)$ and $a \nmid b$ then $a \nmid c$.

3. Prove that if a and b are odd integers then $4 \nmid (a^2 + b^2)$.

4. Prove that the sum of the two legs of a right triangle must be greater than the hypotenuse.

5. Prove that $\sqrt{3}$ is irrational. Just as with our proof with $\sqrt{2}$ you will need a lemma. State and prove this lemma as part of your solution.

6. Prove there does not exist a real number x such that $x^6 + x^4 + 1 = 2x^2$.

7. Prove that the equation $x^3 + x + 1 = 0$ has a real solution but no rational solution.
   Hint: For the second part, if $\frac{p}{q}$ is such a root in lowest terms examine the parities of p and q.

8. Suppose I have a list of real numbers, all between 0 and 1, listed with decimal expansion as follows, with each variable representing a digit:

$$\begin{array}{l} 0.a_{11}a_{12}a_{13}... \\ 0.a_{21}a_{22}a_{23}... \\ 0.a_{31}a_{32}a_{33}... \\ ... \end{array}$$

   Prove there exists a real number not in the list.

9. Prove that there are infinitely many $x, y \in \mathbb{Z}$ with $4x - 6y = 14$.