MATH 310: Homework 7 **Due Thursday 8/2/2012**

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1. Define $A = \{a, b, c, d\}$ and $B = \{x, y, z\}$ and define the relation R from A to B by [5 pts]

$$R = \{(a, x), (a, z), (b, y), (b, z), (c, y)\}$$

   (a) List the elements in $\{\alpha \in A \mid \alpha R y\}$.

   (b) Find $|\{(\alpha, \beta) \mid (\alpha = a) \vee (\beta = z)\}|$.

2. Define a relation R on $\mathbb{Z}$ by $R = \{(a, b) \mid a \leq b + 2\}$. [15 pts]

   (a) Prove or disprove: R is reflexive.

   (b) Prove or disprove: R is symmetric.

   (c) Prove or disprove: R is transitive.

3. Suppose A is the set of all students in this class and we define a relation R on A by sRt if student s has a birthday before or the same day as t. Let x be you. Find the set $\{y \in A \mid xRy\}$. List by first names only. [10 pts]

4. Define a relation R on $\mathbb{Z}$ by $R = \{(a, b) \mid 4|(3a + b)\}$. [20 pts]

   (a) Prove that R is an equivalence relation.

   (b) List the distict equivalence classes for R.

5. Define a relation R on $\mathbb{Z}$ by $R = \{(a, b) \mid a^2 + b^2 \text{ is even}\}$. [20 pts]

   (a) Prove that R is an equivalence relation.

   (b) List the distict equivalence classes for R.

6. Let $A = \{1, 2, 3, 4, 5, 6\}$. Suppose the distict equivalence classes for some relation R are $\{1, 4, 5\}$, $\{2, 6\}$ and $\{3\}$. What is R? [5 pts]

7. A relation R on a nonempty set A is defined to be *circular* if for all $x, y, z \in A$ we have $(xRy \wedge yRz) \rightarrow zRx$. Prove that R is an equivalence relation iff R is circular and reflexive. [20 pts]