MATH 310: Homework 8 **Due Monday 8/6/2012**

1. Find counterexamples to the following false claims:
   (a) In $\mathbb{Z}_{10}$ we have $[2][x] = [6] \;\rightarrow\; [x] = [3]$. [5 pts]
   (b) Delete. [5 pts]
   (c) In $\mathbb{Z}_{15}$ we have $[x][y] = [0] \;\rightarrow\; ([x] = [0] \;\vee\; [y] = [0])$. [5 pts]
2. Write down a multiplication table for the equivalence classes of the integers modulo 5. [10 pts]
3. By passing to equivalence classes modulo a judicious choice of n solve the following problems:
   (a) Show that there are no integer solutions to $21x^3 - 3x^2 + 2x + 8 = 0$. [15 pts]
   (b) Show that if $a \in \mathbb{Z}$ then $a^2 \not\equiv 2 \pmod 4$ and $a^2 \not\equiv 3 \pmod 4$. [15 pts]
4. Consider the equation $6x + 15y = 33$. [20 pts]
   (a) Pass to equivalence classes for some modulus and use this to find a linear expression for x in terms of an unknown integer.
   (b) Do the same for y using a different modulus.
   (c) Plug your answers into the original equation to relate your two unknown integers.
   (d) Give formulas for all solutions to the equation.
5. Prove that $f : (\mathbb{R} - \{1\}) \rightarrow (\mathbb{R} - \{3\})$ given by $f(x) = \frac{3x}{x-1}$ is surjective. [10 pts]
6. Prove that $f : \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = x^3 - 6x^2 + 12x - 7$ is injective. [10 pts]
7. Prove that $f : \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = x^2 - 2x + 4$ is not surjective. [5 pts]
8. Prove that $f : (\mathbb{R} - \{0, 1\}) \rightarrow \mathbb{R}$ given by $f(x) = \frac{1}{x(x-1)}$ is not injective. [5 pts]
9. Give two distinct functions $f_1, f_2 : [0, 1] \rightarrow [0, 1]$ which are both bijective. Prove bijectivity and distinctness. [20 pts]