MATH 310: Homework 9 **Due Thursday 8/9/2012**

1. Consider the function : $(\mathbb{R} - \{3\}) \to (\mathbb{R} - \{1/2\})$ given by $f(x) = \frac{x}{2x-6}$.
   (a) Prove that f is injective. [5 pts]
   (b) Prove that f is surjective. [5 pts]
   (c) Find the algebraic rule for $f^{-1}(y)$. [5 pts]
   (d) Explain what you have shown about the cardinalities of $\mathbb{R} - \{3\}$ and $\mathbb{R} - \{1/2\}$. [5 pts]
   (e) Explain non-rigorously how this idea might be extended to any two sets $\mathbb{R} - \{a\}$ and $\mathbb{R} - \{b\}$ for $a, b \in \mathbb{R}$. [10 pts]
2. Consider the function $f : \mathbb{R} \to [-1, 1]$ given by $f(x) = \sin(x)$.
   (a) Does this function have an inverse? Explain. [5 pts]
   (b) By restricting the domain three different ways find three different inverses of f. You do not need to prove they are inverses but sketch them. [10 pts]
3. Prove that if $f : A \to B$ and $g : B \to C$ are both injective then so is $g \circ f : A \to C$. [5 pts]
4. Prove that $f : \mathbb{N} \to \mathbb{Z}$ given below is a bijection [10 pts]
$$f(n) = \frac{1 + (-1)^n(2n - 1)}{4}$$
5. Suppose A is countably infinite. Prove that $A \times \{1, 2\}$ is also countable. [10 pts]
6. Prove that $|\mathbb{N} \times \mathbb{N}| = |\mathbb{N}|$. [10 pts]