MATH 410 Exam 1 Study Guide

Basic Outline

1.1 Mathematical induction, the Completeness Axiom, infimum, supremum, maximum, minimum, upper bound, lower bound.

1.2 Archimedian Property, dense subset: $\forall (a, b)\ \exists x \in S \cap (a, b)$.

Theorem: $\mathbb{Q}$ and $\mathbb{R} \setminus \mathbb{Q}$ are dense in $\mathbb{R}$.

2.1 Sequences, convergence, Comparison Lemma.

2.2 Bounded sequence, closed subset, sequentially dense: $\forall x_0 \in \mathbb{R}\ \exists \{x_n\}$ in S ct x_0

Theorem: Every convergent sequence is bounded.

2.3 Monotone sequence, Monotone Convergence Theorem, Nested Interval Theorem.

Theorem (MCT): A monotone sequence converges iff it is bounded and if so to sup or inf.

2.4 Sequential compactness: $\forall \{x_n\}$ in $S\ \exists \{x_{n_k}\}$ which converges.

Theorem: Every sequence has a monotone subsequence.

Theorem: Every bounded sequence has a convergent subsequence.

Theorem (SCT): $[a, b]$ is sequentially compact.

3.1 Continuity of a function.

3.2 Extreme Value Theorem. Proof contains some critical techniques.

3.3 Intermediate Value Theorem.

3.4 Uniform Continuity

Theorem: A uniformly continuous function $f : D \to \mathbb{R}$ is continuous.

Theorem: A continuous function $f : [a, b] \to \mathbb{R}$ is uniformly continuous.

3.5 The ϵ-δ criteria.

Theorem: ϵ-δ at x_0 iff continuous at x_0.

Theorem: ϵ-δ on D iff uniformly continuous on D.

3.6 Montone functions, one-to-one, inverses.

Theorem: If $f : D \to \mathbb{R}$ is monotone and $f(D)$ is an interval then f is continuous.

Theorem: If $f : I \to \mathbb{R}$ is strictly monotone then $f^{-1} : f(I) \to \mathbb{R}$ is continuous.

3.7 Limit points, limits of functions.

How to Study

1. Definitions: Practice showing that things either do or do not satisfy definitions. This can be done either straight from the definition or via a theorem. Consider whether definitions are subsets of one another.

2. Theorems: Determine if certain specific examples fit the hypotheses for a theorem and determining the consequences. Determine if the converse of a theorem is true and if not then finding a counterexample. Determine if all hypotheses are required and if not then finding a counterexample.

3. Together: Analyzing how definitions and theorems fit with each other.