MATH 410 Sections 0101,0401: Exam 1 **Friday 3/2/2012**

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1. True or false: Determine if the following are true or false. If false provide a counterexample. [15 pts]
   (a) A subsequence of a monotone sequence is also monotone.
   (b) An subsequence of an unbounded sequence cannot converge.
   (c) A function $f : (1,3) \to \mathbb{R}$ must be bounded.
   (d) A continuous function $f : [a,b] \to \mathbb{R}$ has $f(\mathbb{R})$ bounded.
   (e) A bounded subset of $\mathbb{R}$ must have a maximum.
2. Prove that for $b > 0$ and $n \in \mathbb{N}$ we have $\left(\frac{1}{2} + b\right)^n \geq b^{n-1}\left(b + \frac{1}{2}n\right)$. [10 pts]
3. Using only the Archimedian Principle give a direct proof that $\left\{7 + \frac{1}{n} + \frac{2}{\sqrt{n}}\right\}$ converges to 7. [10 pts]
4. Define $f : \mathbb{R} \to \mathbb{R}$ by $f(x) = x^2 + 2$. Verify the ϵ-δ criterion for $f(x)$ at $x = 3$. [15 pts]
5. Prove that the function [15 pts]
$$f(x) = \begin{cases} x^2 & \text{if } x < 1 \\ x - 1 & \text{if } x \geq 1 \end{cases}$$
   is continuous at $x = 2$ but not at $x = 1$.
6. Prove that the function $f : (0,1) \to \mathbb{R}$ given by $f(x) = \frac{1}{x-1}$ is not uniformly continuous. [10 pts]
7. Prove that a nonnegative convergent sequence must converge to a nonnegative value. [10 pts]
8. Suppose $\{b_n\}$ is a bounded nonnegative sequence and $0 \leq r < 1$. Define [15 pts]
$$s_n = b_1 r + b_2 r^2 + b_3 r^3 + ... + b_n r^n$$
   Prove that $\{s_n\}$ converges. Hint: Use the Monotone Convergence Theorem.

The End