1. True or false: Determine if the following are true or false. If false provide a counterexample. You do not need to prove anything about the counterexamples, just provide them. [20 pts]
   (a) An integrable function must be continuous.
   (b) If $f : [a, b] \to \mathbb{R}$ is continuous then it has an antiderivative.
   (c) If $f : [0, 2] \to \mathbb{R}$ is integrable on every open interval inside $[0, 2]$ then it is integrable on $[0, 2]$.
   (d) If $f, g : D \to \mathbb{R}$ are both differentiable on D and have the same derivative then f and g differ by a constant on D.
   (e) If $f : (a, b) \to \mathbb{R}$ is differentiable then f is continuous.

2. Define $f(x) = \frac{1}{x}$. Use the limit definition of the derivative and the sequence definition of a limit to find $f'(2)$. [10 pts]

3. Find a formula not involving integrals for $F(x) = \int_0^x f$ and explain how you know that F is an antiderivative of f for $f : [0, 4] \to \mathbb{R}$ defined by [10 pts]

$$f(x) = \begin{cases} x & \text{if } x \in [0, 2] \\ 2 - x & \text{if } x \in (2, 4] \end{cases}$$

4. Use the Archimedes-Riemann Theorem to find $\int_0^3 x\ dx$. [10 pts]

5. Suppose $f : (0, 4) \to \mathbb{R}$ has $f(2) = 7$, $f'(2) = 0$, $f''(2) = 0$ and $f'''(x) > \frac{1}{2}$ for $x \in (0, 4)$. Find a lower bound on $f(3.9)$. [10 pts]
   Note: If you're not sure how to handle the 7 you can replace it by 0 for a maximum of 8/10 points.

6. Prove that if $f : [a, b] \to \mathbb{R}$ is monotonically increasing then f is integrable. [10 pts]
   Note: You may not use the theorem that says monotonically increasing functions are integrable!

7. Suppose $h : \mathbb{R} \to \mathbb{R}$ is bounded. Define $f : \mathbb{R} \to \mathbb{R}$ by [15 pts]

$$f(x) = 1 + 4x + x^2 h(x)$$

   Prove that $f(0) = 1$ and $f'(0) = 4$.
   Note: You may not assume that h is differentiable.

8. Suppose $f : \mathbb{R} \to \mathbb{R}$ is differentiable at $x_0 \in (a, b)$ and $f'(x_0) > 0$. Prove there is a neighborhood I of x_0 such that for $x \in I$ we have $x < x_0 \Rightarrow f(x) < f(x_0)$ and $x > x_0 \Rightarrow f(x) > f(x_0)$. [15 pts]

The End