MATH 410: Homework 1 **Due in class Friday 9/7/2012**

1. Explain in full sentences how the method of proof by contradiction would be applied to prove that $P \to Q$. *

2. Negate each of the following quantified statements, passing the negation through as far as possible.

   (a) $\forall x \in \mathbb{R}, P(x)$ *

   (b) $\exists x \in \mathbb{Z}, (Q(x) \wedge R(x))$ *

   (c) $\forall \epsilon \in \mathbb{R}^+, \exists n \in \mathbb{N}, P(\epsilon, n)$ *

   (d) $\exists x \in \mathbb{R}, \forall y \in \mathbb{R}, (P(x,y) \to Q(x,y))$ *

3. Fill in the following table. If a set does not have the entry specified write "None". **
   No proof is necessary.

Set	Inf	Min	Max	Sup
$[0,2)$				
$\{x \in \mathbb{Q} \mid x \geq \sqrt{3}\}$				
$[-1,1] \cap \mathbb{Q}$				
$\{x \mid x < x^2\}$				
$\{..., -3, -2, -1, 0, 1\}$				
$\{0.5^n \mid n \in \mathbb{N}\}$				

4. Consider the set $S = \{x \in \mathbb{R} \mid x^2 < 2\}$.

   (a) Prove that $\sqrt{2}$ is an upper bound for S. *

   (b) Prove by contradiction that $\sqrt{2}$ is the least upper bound for S. **

5. Prove that if $S \subseteq T \subseteq \mathbb{R}$ and if T is bounded above then $\sup S \leq \sup T$. *

6. Suppose that S is a bounded nonempty set of real numbers. Prove that $\inf S \leq \sup S$. *

7. Which of the following is true and which is false. Provide proof.

   (a) If $S \subseteq \mathbb{R}$ has no maximum then for all $x \in \mathbb{R}$ we can find some $s \in S$ with $s > x$. *

   (b) If $S \subseteq \mathbb{R}$ has no supremum then for all $x \in \mathbb{R}$ we can find some $s \in S$ with $s > x$. *